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Recently, the number of multiple authorship and collaborative papers has been growing rapidly. This number differs significantly according to various scientific fields. Known that *h*-type indices (*h*-index, *g*-index, *A*-index, etc.) are used to evaluate the performance of researchers, which do not distinguish between single-author and multi-author papers in the evaluation process. In other words, a citation received from multi-authored papers is applied to all co-authors (as in the single-authored paper). To solve this problem, several weighted version of the *h*-index have been proposed. Most of these versions are indices and are based on the division principle of citations based on the co-author's position in the authors' order. In other words, according to the position of the co-authors order, the weight is assigned to the co-authors, and the citations are proportionally divided according to co-authors in these weights. Obviously, the calculation of weights is important in this case. h- type indices proposed in the paper are based on the "relative first author" (or "local first author") principle. "Relative first author" means the co-authors being in the first position relative to the co-authors after him. Based on this principle, existing weighting schemes were modified, and then new weighted *h*- type indices were proposed for multiple authorship papers according to these weighting schemes. In other words, the "local first author" approach was proposed instead of the "global first author" approach in calculating the h- type indices for multiple authorship papers. The suggested indices were calculated for 30 researchers selected from the Google Scholar database and compared with other relevant *h*-type indices.

Keywords: *Multiple authorship, h-index, Relative proportionally weighted h-index, Relative geometric weighted h-index, Relative harmonic weighted h-index, g-index, Relative proportionally weighted g-index, Relative harmonic weighted h-index, Relative geometric weighted g-index.*

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1. Introduction

Science is growing exponentially and doubles in size every according to Price law, 10-15 years (Price, 1951, 1961, 1965). The exponential growth in science has also been confirmed by other studies (Tabah, 1999). In the context of scientific information abundance, the calculation of researchers' performance is important today and probably will be the same in the following years. The researcher's performance can be assessed based on a number of criteria. These criteria include scientific, practical, innovative, pedagogical, and other activities. As a rule, published papers and citations can be taken as a key to the result of scientific activity. In this paper, the performance of the researcher is evaluated on the basis of these two indicators (number of papers and citations). It is known that the first index taking into account both indicators when performance evaluating the researcher is the ***h*-index** (Hirsch, 2005). Simple of calculations can be accepted as the advantage of the *h*-index. Shortcomings such as management through self-citation, not distinguishing scientific fields, also single and multiple authorship papers; not considering the author's contribution to the paper caused the development of other modified versions of the *h*-index (Egghe, 2006; Shreiber, 2008, 2009; Bornmann et al., 2011).

Recently, the increase of papers written in collaboration, in other words, the number of multiple authorship papers, made it necessary to consider the multiple authorship in the *h*-index. Thus, applying citation to all authors in multiple-authorship papers, as in solo-authorship papers, has raised some questions. The main question is the distribution principle of citations between co-authors of multiple-authorship papers. In recent years, researchers have proposed several weighted versions of the *h*-index to solve this problem (Abbas, 2011; Shreiber, 2008; Hagen, 2013; Zang, 2009). In all these approaches, the contribution of each co-author is determined by his or her position in the list of authors and the length of that list». Weights are assigned to co-authors according to their position, and citations are distributed among them in proportion to these weights. The studies show (Abbas, 2011; Hagen, 2008, Oppenheim, 1998, Shreiber, 2008) that in such approaches, the first author is assigned too much weight, and consequently, most of the citations are attributed to the first author. In these schemes, the difference between the weight of each author and the weight of the next author is large. The paper first introduces the relative first author principle, and then existing weight schemes are modified according to this «relative first author» principle. Finally, new types of *h*- and *g*-indices (local proportional, local geometric and local harmonic) are proposed.

2. Related Work

Hirsch (2005) proposed the ***h*-index** to evaluate the performance of researchers. He noted that this index allows us to evaluate the importance and impact of researcher contribution and is easy to calculate. This index can also be accepted as an essential criterion for evaluating scientific achievements and comparing different scientists.

A scientist has index h if h of his/her N_p *papers have at least h citations each and the other* $(N_p - h)$ *papers have ≤ h citations each.* The definition of the h-index can be expressed by Eq.(1):

$$h = \max\{h \mid c_h \geq h\} \tag{1}$$

h-index is used not only for the evaluation of scientists, but also for the evaluation of journals, organizations and countries.

Most criticized shortcomings of h-index are that it does not distinguish between a novice researcher and a researcher who has been engaged in research for many years, the h-index management by self-citation, not consider the research field, not distinguish between single-authored and multi-authored papers and, also do not consider the authors' contributions to the paper. To overcome these shortcomings, scientists have proposed different modifications to the h-index. A brief review of these modifications is given below.

The concept of assigning weight (credit) to co-authors depending on their position in the author list was proposed by researchers Lindsey (1980), Hodge and Greenberg (1981), Van Hooydonk, 1997, Trueba & Guerrero (2004). Lindsey (1980) proposed an accounting procedure for measuring production, which involves dividing the paper produced by the number of authors. To allocate authorship credit for multi-authored publications according to a harmonic progression was originally suggested by Hodge and Greenberg (1981) in a letter to Science. Their letter was a response to Derek De Solla Price who, although aware that co-authors did not contribute equally, had proposed equal division of publication and citation credit among co-authors as "a deterrent to the otherwise pernicious practice of coining false brownie points by awarding each author full credit for the whole thing" (Price 1981).

Thus, they suggested assigning weight to the authors based on three principles: (**1**) the value or significance of a given publication should be shared among all its authors; (**2**) total publication credit should be divided among authors; (**3**) first authors should be credited more than later authors in the same paper. The proposed approach by them can be used to assess scientific productivity, the impact of the scientist on a particular scientific field.

Wan et al. (2007) suggested the positive aspects of h-index such as being simple, that uncited papers not affecting to the h-index, etc. However, the shortcoming of this index was not considered suitable index for beginners in the scientific field, the increasing of h-index while the number of citations to previous papers increases, difficulty to compare the scientists in different fields, etc.

g-index is one of the modified types of the h-index proposed by Egghe (2006). *If the most cited g papers of researcher have at least* g^2 *citations in total, then it has a g-index.* The g-index is defined as follows:

$$g = \max\{i \mid \textstyle\sum_{j=1}^{i} c_j \geq i^2\} \tag{2}$$

The g-index considers the paper citations that exceed h, is accepted as its advantage.

Tscharntke et al. (2007) studied the credit (weight) assigning problem, considering the sequence of authors and their contribution to the paper. They noted that traditionally, the first author receives more credit for the contribution of the papers than the others, but other authors' contributions are determined by their alphabetical order or reverse seniority. Their approach is as follows:

1. **The "sequence-determines-credit" approach (SDC):** the sequence of authors should be considered according to descending order of their contribution. The first author takes the whole weight, the second author half of the weight, the third author 1/3 of the weight, and so on.
2. **The "equal contribution" norm (EC)**: the equal distribution of weight among authors.
3. **The "first-last-author-emphasis" norm (FLAE) (first and last author):** the first author takes total weight, the last author half of the weight, and the others take the relative weight to the total number of authors.
4. **The "percent-contribution-indicated" approach (PCI):** estimate the percentage of each author's contribution

Shreiber (2008, 2009) proposed h_m-index (h-multiple authorship) for multiple-authorship papers. h_m-index which is determined in analogy to the h-index, but counting the papers fractionally according to the number of authors, for example, only as one third for three authors.

Hu et al. (2010) proposed the h-major index which considers the authors main and co-authorial role in multiple-authorship papers. h-maj, which takes only those articles into account in which the scientist plays a major or core role.

Bornmann et al. (2011) firstly studied the meta-analysis issue of 37 various modifications of h-index, gave a brief review about the existed indices, and determined the h_i, h_m, h_{ms}, and h-maj indices as the multi-authorship indices.

The h_w-index and the h_{wt}-index were proposed by Feng & Mo (2019). h_w-index calculates the researcher's contribution to each paper, considering the highly cited papers. Oppose to h -index, the h_w-index evaluates the researcher's contribution to the paper easier. h_{wt}-index focuses on the researcher's recent papers. This index is considered to be more suitable for the evaluation of young researchers because the evaluation time is taken for a short period.

As seen, scientists have proposed different modifications to address the shortcomings of the h-index, including the problem of ignoring of the authors' contribution.

The h-index is based on evaluation of weighted citation for multiple-authorship papers. Let C_p be number of citations of p-th paper of an author, the weighted number of citations C_p^w of the pth paper is as follows:

$$C_p^w = C_p w(p) \tag{3}$$

where $w(p)$ is the weight assigned to the given author for his/her p-th paper under a weight assignment scheme. For simplicity, will be used w instead of notation $w(p)$.

In the paper, various schemes have been used to evaluate weighted citation. Brief review of these schemes is given below.

Fractional weighting (Price, 1981; Oppenheim, 1998) scheme assigns to each author a score equal to $\frac{1}{n}$:

$$w_i^{\mathrm{f}} = \frac{1}{n},\ i = 1,\ldots,n \tag{4}$$

where i is the rank (position) of an author in the author list, n is the number of co-authors.

Proportional weighting (Van Hooydonk, 1997; Abbas, 2011) assigns co-authors proportional weight depending on their position in the author list. If an author has rank i in the author list of a paper with n authors, then she/he receives a weight of $(n + 1 - i)$. This weight can be normalized in such a way that the total weight of all authors is equal to 1. In this normalized version the weight is (Van Hooydonk, 1997):

$$w_i^p = \frac{2(n+1-i)}{n(n+1)},\ i = 1,\ldots,n \tag{5}$$

Geometric weighting (Egghe et al., 2000). If an author has rank i in a paper with n co-authors then she/he receives a credit of:

$$w_i^{\mathrm{g}} = \frac{2^{n-i}}{2^n - 1},\ i = 1,\ldots,n \tag{6}$$

Harmonic weighting (Hagen, 2008, 2013). If an author has rank i in a paper with n co-authors then she/he receives a credit of:

$$w_i^{\mathrm{h}} = \frac{1/i}{1+1/2+\ldots+1/n},\ i = 1,\ldots,n \tag{7}$$

The Eq.(7) can be written in a compact form as:

$$w_i^h = \frac{1}{iH_n} \tag{8}$$

where

$$H_i = \sum_{j=1}^{i} \frac{1}{j};\ i = 1,\ldots,n. \tag{9}$$

Zhang (2009) proposed the following Eq.(10) to evaluate the author credit in the i-th position in four or more co-authored papers:

$$w_i^n = \frac{2(n+1-i)}{(n+1)(n-2)},\ n \geq 4,\ 2 \leq i \leq n-1 \tag{10}$$

After analysis the *h*-index and its modified variants we conclude that there doesn't exist a unified scheme for evaluation of authors' contribution in multiple-authored papers. Its obvious that in all weighting schemes the first author carries the greatest weight in comparison with others. In the proposed approach considering that the first-author concept is absolute, each author is accepted as the first in relation to subsequent authors. The replacement of the first author notion (in other words, global first author) with the notion of the **relative first author** (local first author) is recommended. Relative (local) first author means that in multiple-authorship papers, each author is in the first position compared to the author(s) after him/her in the authors list. The advantage of the proposed approach is to decrease the difference between the weight of the neighbour authors as much as possible. This allows to avoid the first-author to carry great weight in the previous schemes. The proposed weighting schemes are described in detail in the next section.

3. Proposed Weighting Schemes

As in the proposed approaches, the first author has a greater weight than the others. In this section, the assignment of three different weights to the authors has been suggested depending on their position in multiple authorship papers: relative-proportional weighted *h*-index w_i^{rp}, relative geometric weighted *h*-index w_i^{rg}, and relative harmonic weighted *h*-index w_i^{rh}. In the paper, the notion of the first author is replaced by the notion of the "relative" first author. Here, each author is considered **the first author** in respect of his next co-authors. In other words, it can be called a relative first position. Every position is the first according to the next co-authors(s) all following. For example, in the four-authored paper, the second author is considered the first author according to the third one, and the third author is considered the first author according to the fourth one.

The proposed relative proportional w_i^{rp}, relative geometric w_i^{rg} and relative harmonic w_i^{rh} weights are defined by Eqs.(11), (23) and (25), respectively.

Definition 1 (*relative proportional weight*) : If an author has rank *i* in a paper with *n* co-authors then she or he receives a credit of:

$$w_i^{\mathrm{rp}} = \frac{1}{(i+1)\cdot P_n},\ i = 1,\ldots,n \tag{11}$$

$$P_n = \sum_{j=1}^{n} \frac{1}{j+1} = H_n - \frac{n}{n+1} \tag{12}$$

The obtaining scheme of Eq.(11) is given below step-by-step.

Let the number of co-authors is *n*. Then, we point out the co-author's list by $A_1 A_2 A_3 \cdots A_{n-1} A_n$.

Step 1. The proportional weight of the 1st author (A_1) in the $A_1 A_2 A_3 \cdots A_{n-2} A_{n-1} A_n$ list of the co-authors is evaluated using the Eq.(5):

$$w_1^{\mathrm{p}}(A_1) = \frac{2(n+1-1)}{n(n+1)} = \frac{2}{n+1} \tag{13}$$

Step 2. The first author (A_1) is deleted from the list of $A_1 A_2 \cdots A_{n-1} A_n$ and the proportional weight of the first author (A_2) among the new co-authors is evaluated (in this case the number of co-authors is $n-1$):

$$w_1^{\mathrm{p}}(A_2) = \frac{2(n-1)}{n(n-1)} = \frac{2}{n} \tag{14}$$

Step 3. The 1st author (A_2) is again deleted from the $A_2 \cdots A_{n-1} A_n$ list of co-authors, and the proportional weight of the 1st author (A_3) is evaluated among the newly acquired $A_3 \cdots A_{n-1} A_n$ co-authors list. In this case, as the number of co-authors is (n – 2), so the weight will be as follows according to Eq.(5):

$$w_1^{\mathrm{p}}(A_3) = \frac{2(n-2)}{(n-2)(n-1)} = \frac{2}{n-1} \tag{15}$$

Step 4. Continuing the process of removing the 1st author in this way, we evaluate the proportional weight of the author (A_{n-2}), who is still in the 1st position among the, $A_{n-2} A_{n-1} A_n$ list of co-authors (number of co-authors = 3):

$$w_1^{\mathrm{p}}(A_{n-2}) = \frac{2(3+1-1)}{3(3+1)} = \frac{2}{4} \tag{16}$$

Step 5. The proportional weight of A_{n-1} which is in the first position in the $A_{n-1} A_n$ list of co-authors will be

$$w_1^{\mathrm{p}}(A_{n-1}) = \frac{2(2+1-1)}{2(2+1)} = \frac{2}{3} \tag{17}$$

after removing of A_{n-2} from the authors' list $A_{n-2} A_{n-1} A_n$ (the number of co-authors = 2).

Step 6. Finally, the weight will be

$$w_1^{\mathrm{p}}(A_n) = \frac{2(1+1-1)}{1(1+1)} = \frac{2}{2} \tag{18}$$

for the A_n after removing A_{n-1} from the list of $A_{n-1} A_n$.

Step 7. After the ending of the process, the relative proportional weights of the authors are evaluated. The following Eq.(19) is proposed to calculate this weight:

$$w_i^{\mathrm{rp}} = w_1^{\mathrm{rp}}(A_i) = \frac{w_1^{\mathrm{p}}(A_{n+1-i})}{\sum_{k=1}^{n} w_1^{p}(A_k)},\ i = 1, \cdots, n \tag{19}$$

Where

$$\sum_{k=1}^{n} w_1^{\mathrm{p}}(A_k) = \frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \cdots + \frac{2}{n-1} + \frac{2}{n} + \frac{2}{n+1} = 2\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n-1} + \frac{1}{n} + \frac{1}{n+1}\right)$$
$$= 2\sum_{j=1}^{n} \frac{1}{j+1} = 2\left(H_n - \frac{n}{n+1}\right) = 2P_n \tag{20}$$

$$w_1^{\mathrm{p}}(A_{n+1-i}) = \frac{2}{i+1},\ i = 1, \cdots, n \tag{21}$$

Now, taking into account the Eqs, (20) and (21), we obtain the Eq.(11):

$$w_i^{\mathrm{rp}} = w_1^{\mathrm{rp}}(A_i) = \frac{2 \cdot \frac{1}{i+1}}{2P_n} = \frac{1}{(i+1) \cdot P_n},\ i = 1, \cdots, n \tag{22}$$

Definition 2 (*relative geometric weight*). If an author has rank *i* in a paper with *n* co-authors then she/he receives a following credit of:

$$w_i^{\mathrm{rg}} = \frac{2^{i-1}}{(2^i - 1)\cdot G_n},\ i = 1,\ldots,n \tag{23}$$

where

$$G_n = \sum_{j=1}^{n} \frac{2^{j-1}}{2^j - 1} \tag{24}$$

Definition 3 (*relative harmonic weight*). If an author has rank i in a paper with n co-authors then she/he receives a following credit of:

$$w_i^{\mathrm{rh}} = \frac{1}{H_i \cdot H_n^{(-1)}} \tag{25}$$

$$\mathrm{H}_n^{(-1)} = \sum_{j=1}^{n} \frac{1}{\mathrm{H}_j} \tag{26}$$

Note. Eqs.(23) and (25) are obtained analogiously, according to the above scheme. The only difference is in the calculation of the geometric Eq.(6) and harmonic Eq.(7) weights for the 1st author at each step, respectively.

4. Comparison of Weighting Schemes

In this section we compare the proposed weighting schemes with other weighting schemes.

Weights ratio of neighboring co-authors. The weight ratios of the i-th and (i+1)-th authors are as follows:

$$\frac{w_i^{\mathrm{f}}}{w_{i+1}^{\mathrm{f}}} = \frac{\frac{1}{n}}{\frac{1}{n}} = 1 \tag{27}$$

In other words, the weights ratio of neighboring authors in the fractional scheme is equal to 1.

In the global proportional scheme, the weight ratio of neighboring authors depends on both the number of authors n and the position i:

$$\frac{w_i^{\mathrm{p}}}{w_{i+1}^{\mathrm{p}}} = \frac{\frac{2(n-i+1)}{n(n+1)}}{\frac{2(n-i-1+1)}{n(n+1)}} = \frac{2(n-i+1)}{2(n-i)} = \frac{n-i+1}{n-i} = 1 + \frac{1}{n-i} \tag{28}$$

However, in the proposed local (relative) proportional scheme, the weight ratio of neighboring authors depends only on his/her position in the authors list:

$$\frac{w_i^{\mathrm{rp}}}{w_{i+1}^{\mathrm{rp}}} = \frac{\frac{1}{(i+1)P_n}}{\frac{1}{(i+1+1)P_n}} = \frac{i+2}{i+1} = 1 + \frac{1}{i+1} \tag{29}$$

In the geometric scheme, the weight ratio of neighboring authors is constant, i.e. equal to 2:

$$\frac{w_i^{g}}{w_{i+1}^{g}}=\frac{\frac{2^{n-i}}{2^n-1}}{\frac{2^{n-i-1}}{2^n-1}}=\frac{2^{n-i}}{2^n-1}\cdot\frac{2^n-1}{2^{n-i-1}}=2 \tag{30}$$

In the local geometric scheme, this ratio depends on the position of the author and is less than 2:

$$\frac{w_i^{rg}}{w_{i+1}^{rg}}=\frac{2^{i-1}}{2^i-1}\cdot\frac{2^{i+1}-1}{2^i}=\frac{2^{i+1}-1}{2(2^i-1)}=\frac{2^{i+1}-1}{2^{i+1}-2}=\frac{2^{i+1}-2+1}{2^{i+1}-2}=1+\frac{1}{2(2^i-1)}=1+\frac{1}{2^{i+1}-2} \tag{31}$$

In the global and local harmonic schemes, the ratio of the weights of neighboring authors depends only on the author position *i*:

$$\frac{w_i^{h}}{w_{i+1}^{h}}=\frac{\frac{1}{iH_n}}{\frac{1}{(i+1)H_n}}=\frac{i+1}{i}=1+\frac{1}{i} \tag{32}$$

$$\frac{w_i^{rh}}{w_{i+1}^{rh}}=\frac{\frac{1}{H_iH_n^{(-1)}}}{\frac{1}{H_{i+1}H_n^{(-1)}}}=\frac{H_{i+1}}{H_i}=1+\frac{1}{(i+1)H_i} \tag{33}$$

As seen the weights ratios of the neighboring authors are relatively small in the local harmonic scheme compared to the global harmonic scheme.

Summarizing the above mentioned, we can conclude that the weight ratio of neighboring authors in the proportional scheme depends on both the number of authors *n*, and the position (*i*). For other weighting schemes this ratio depends only on the position (*i*).

Difference of weights of neighboring co-authors. The difference is calculated as follows:

$$w_i^{f}-w_{i+1}^{f}=\frac{1}{n}-\frac{1}{n}=0 \tag{34}$$

The difference of weights of two neighboring authors for the fractional scheme is equal to 0.

In the global proportional scheme, the weight difference depends on the number of authors, while in the local proportional scheme, the difference depends on both the author's position and the number of authors:

$$w_i^{p}-w_{i+1}^{p}=\frac{2(n-i+1)}{n(n+1)}-\frac{2(n-i-1+1)}{n(n+1)}=\frac{2}{n(n+1)} \tag{35}$$

$$w_i^{rp}-w_{i+1}^{rp}=\frac{1}{(i+1)P_n}-\frac{1}{(i+1+1)P_n}=\frac{1}{(i+1)(i+2)P_n} \tag{36}$$

The same situation is observed for the geometric, relative geometric, harmonic, and relative harmonic schemes:

$$w_i^{\mathrm{g}} - w_{i+1}^{\mathrm{g}} = \frac{2^{n-i}}{2^n-1} - \frac{2^{n-i-1}}{2^n-1} = \frac{2^{n-i}}{2^n-1}(1-2^{-1}) = \frac{2^{n-i-1}}{2^n-1} \tag{37}$$

$$\begin{aligned} w_i^{\mathrm{rg}} - w_{i+1}^{\mathrm{rg}} &= \frac{1}{G_n}\left(\frac{2^{i-1}}{(2^i-1)} - \frac{2^i}{2^{i+1}-1}\right) = \frac{1}{G_n}\left(\frac{2^i}{2(2^i-1)} - \frac{2^i}{2^{i+1}-1}\right) = \frac{2^i}{G_n}\left(\frac{1}{2^{i+1}-2} - \frac{1}{2^{i+1}-1}\right) \\ &= \frac{2^i}{G_n}\frac{2^{i+1}-1-2^{i+1}+2}{2(2^i-1)(2^{i+1}-1)} = \frac{1}{G_n}\frac{2^{i-1}}{(2^i-1)(2^{i+1}-1)} \end{aligned} \tag{38}$$

$$w_i^{\mathrm{h}} - w_{i+1}^{\mathrm{h}} = \frac{1}{iH_n} - \frac{1}{(i+1)H_n} = \frac{1}{i(i+1)H_n} \tag{39}$$

$$w_i^{\mathrm{rh}} - w_{i+1}^{\mathrm{rh}} = \frac{1}{iH_n} - \frac{1}{(i+1)H_n} = \frac{1}{(i+1)H_i H_{i+1} H_n^{(-1)}} \tag{40}$$

From Eq.(35) follows that in the proportional weighting scheme, the difference in weights of neighboring authors depends only on the number of authors (n). From Eqs. (36)-(40) we can see that for other schemes this difference also depends on the position, i.

The difference between proportional and fractional weights is:

$$w_i^{\mathrm{p}} - w_i^{\mathrm{f}} = \frac{2(n-i+1)}{n(n+1)} - \frac{1}{n} = \frac{2n-2i+2-n-1}{n(n+1)} = \frac{n+1}{n(n+1)} - \frac{2i}{n(n+1)} = \frac{1}{n} - \frac{2i}{n(n+1)} \tag{41}$$

Weights ratio of the first and last co-authors. The ratio of the fractional weights of the first and last co-authors is equal to 1:

$$\frac{w_1^{\mathrm{f}}}{w_n^{\mathrm{f}}} = \frac{\frac{1}{n}}{\frac{1}{n}} = 1 \tag{42}$$

The ratio of the proportional and relative proportional weights of the first and last co-authors is:

$$\frac{w_1^{\mathrm{p}}}{w_n^{\mathrm{p}}} = \frac{\frac{2n}{n(n+1)}}{\frac{2}{n(n+1)}} = \frac{2n}{n(n+1)} \cdot \frac{n(n+1)}{2} = n \tag{43}$$

$$\frac{w_1^{\mathrm{rp}}}{w_n^{\mathrm{rp}}} = \frac{\frac{1}{2P_n}}{\frac{1}{(n+1)P_n}} = \frac{1}{2P_n} \cdot (n+1)P_n = \frac{n+1}{2} \tag{44}$$

The ratio between the weights of the first and last authors in the global and local proportional scheme depends on the number of authors (n), but there is twice the difference between the weights of the global proportional scheme and the local proportional scheme.

The ratio of the geometric and relative geometric weights of the first and last co-authors is as follows:

$$\frac{w_1^{\mathrm{g}}}{w_n^{\mathrm{g}}} = \frac{\frac{2^{n-1}}{2^{n-1}}}{\frac{2^{n-n}}{2^{n-1}}} = \frac{2^{n-1}}{2^n-1} \cdot \frac{2^n-1}{1} = 2^{n-1} \tag{45}$$

$$\frac{w_1^{\mathrm{rg}}}{w_n^{\mathrm{rg}}} = \frac{\frac{1}{G_n}}{\frac{2^{n-1}}{2^{n-1}}} \cdot \frac{1}{nG_n} = \frac{2^{n-1}}{2^{n-1}} = \frac{2 \cdot 2^{n-2}}{2^n} = 2 - \frac{1}{2^{n-1}} \tag{46}$$

As seen, while the number (n) of authors in the global geometric weight scheme increases in the interval [1, 2), the weight ratio of the first and last (n-th) authors increases very rapidly. In the local geometric weight scheme, this ratio varies in the intervals [1, 2), and as the number of authors increases, it approaches 2. For example, for $n = 10$, these ratios will be 512 and 1.998, respectively.

The ratio of the harmonic weights of the first and last co-authors is:

$$\frac{w_1^{\mathrm{h}}}{w_n^{\mathrm{h}}} = \frac{\frac{1}{H_n}}{\frac{1}{nH_n}} = n \tag{47}$$

This means that the weight of the first author is n times greater than the weight of the last author. In relative harmonic weights, this ratio is as follows:

$$\frac{w_1^{\mathrm{rh}}}{w_n^{\mathrm{rh}}} = \frac{H_n}{H_1} = H_n \approx \ln(n) + \gamma \tag{48}$$

where $\gamma = 0.577$ is the Euler–Mascheroni constant.

The calculations show that in all schemes (except the fractional scheme) the ratio of the first and last (n-th) co-authors weights depends only on the number of co-authors, n. For the fractional scheme, this difference is constant, equal to 1.

Ratio of weights ratio of neighboring co-authors. The ratio of the proportional weights of two consecutive co-authors to the proportional weights of the next two consecutive co-authors is as follows:

$$\frac{w_i^{\mathrm{p}}}{w_{i+1}^{\mathrm{p}}} \div \frac{w_{i+1}^{\mathrm{p}}}{w_{i+2}^{\mathrm{p}}} = \frac{(n-i+1)}{(n-i)} \div \frac{(n-i)}{(n-i-1)} = \frac{(n-i)^2-1}{(n-i)^2} = 1 - \frac{1}{(n-i)^2} < 1 \tag{49}$$

As seen, this ratio is less than 1. However, for other schemes, this ratio is greater than 1:

$$\frac{w_i^{\mathrm{rp}}}{w_{i+1}^{\mathrm{rp}}} \div \frac{w_{i+1}^{\mathrm{rp}}}{w_{i+2}^{\mathrm{rp}}} = \frac{(i+2)}{(i+1)} \div \frac{(i+3)}{(i+2)} = \frac{(i+2)^2}{(i+3)((i+1)} = \frac{i^2+4i+4}{i^2+4i+3} = 1 + \frac{1}{i^2+4i+3} > 1 \tag{50}$$

$$\frac{w_i^{\mathrm{h}}}{w_{i+1}^{\mathrm{h}}} \div \frac{w_{i+1}^{\mathrm{h}}}{w_{i+2}^{\mathrm{h}}} = \frac{\frac{i+1}{i}}{\frac{i+2}{i+1}} = \frac{(i+1)^2}{i^2+2i} = \frac{i^2+2i+1}{i^2+2i} = 1 + \frac{1}{i^2+2i} > 1 \tag{51}$$

$$\frac{w_i^{\mathrm{rh}}}{w_{i+1}^{\mathrm{rh}}} \div \frac{w_{i+1}^{\mathrm{rh}}}{w_{i+2}^{\mathrm{rh}}} = \frac{H_{i+1}}{H_i} \div \frac{H_{i+2}}{H_{i+1}} = \frac{H_{i+1}^2}{H_i H_{i+2}} = \frac{H_{i+1}^2}{\left(H_{i+1} - \frac{1}{i+1}\right) H_{i+1} + \frac{i}{i+2}} = \frac{H_{i+1}^2}{H_{i+1}^2 - \frac{1+H_{i+1}}{(i+1)(i+2)}} > 1 \tag{52}$$

$$\begin{aligned}\frac{w_i^{\mathrm{rg}}}{w_{i+1}^{\mathrm{rg}}} \div \frac{w_{i+1}^{\mathrm{rg}}}{w_{i+2}^{\mathrm{rg}}} &= \frac{\frac{2^{i-1}}{(2^i-1)G_n}}{\frac{2^i}{(2^{i+1}-1)G_n}} \div \frac{\frac{2^i}{(2^{i+1}-1)G_n}}{\frac{2^{i+1}}{(2^{i+2}-1)G_n}} = \frac{2^{i+1}-1}{2^{i+1}-2} \div \frac{2^{i+2}-1}{2^{i+2}-2} = \frac{(2^{i+1}-1)(2^{i+2}-2)}{(2^{i+1}-2)(2^{i+2}-1)} = \frac{2(2^{i+1}-1)(2^{i+1}-1)}{2(2^i-1)(2^{i+2}-1)} \\ &= \frac{2^{2i+2}-2^{i+2}+1}{2^{2i+2}-2^i-2^{i+2}+1} = \frac{(2^{2i+2}-2^i-2^{i+2}+1)+2^i}{2^{2i+2}-2^i-2^{i+2}+1} = 1 + \frac{2^i}{2^{2i+2}-2^i-2^{i+2}+1} \\ &= 1 + \frac{1}{4\cdot 2^i - 5 + 1/2^i} > 1\end{aligned} \tag{53}$$

In the geometric weighting scheme, this ratio is constant, equal to 1:

$$\frac{w_i^{\mathrm{g}}}{w_{i+1}^{\mathrm{g}}} \div \frac{w_{i+1}^{\mathrm{g}}}{w_{i+2}^{\mathrm{g}}} = 2 \div 2 = 1 \tag{54}$$

Below we also define, how increasing the number of co-authors does influence on the weights change for each scheme:

$$w_i^{\mathrm{p}}(n) : w_i^{\mathrm{p}}(n+1) = \frac{2(n+1-i)}{n(n+1)} : \frac{2(n+2-i)}{(n+1)(n+2)} = \frac{(n+1-i)}{(n+2-i)} \frac{n+2}{n} = \left(1+\frac{2}{n}\right)\left(1-\frac{1}{n+2-i}\right) \tag{55}$$

$$\begin{aligned}w_i^{\mathrm{rp}}(n) : w_i^{\mathrm{rp}}(n+1) &= \frac{1}{(i+1)P_n} : \frac{1}{(i+1)P_{n+1}} = \frac{P_{n+1}}{P_n} = \frac{H_{n+1} - \frac{n+1}{n+2}}{H_n - \frac{n}{n+1}} = \frac{H_n + \frac{1}{n+1} - \frac{n+1}{n+2}}{H_n - \frac{n}{n+1}} = \frac{\sum_{j=1}^{n+1} \frac{1}{j+1}}{\sum_{j=1}^{n} \frac{1}{j+1}} \\ &= \frac{\sum_{j=1}^{n} \frac{1}{j+1} + \frac{1}{n+2}}{\sum_{j=1}^{n} \frac{1}{j+1}} = 1 + \frac{\frac{1}{n+2}}{\sum_{j=1}^{n} \frac{1}{j+1}} = 1 + \frac{1}{(n+2)\sum_{j=1}^{n} \frac{1}{j+1}}\end{aligned} \tag{56}$$

$$w_i^{\mathrm{g}}(n) : w_i^{\mathrm{g}}(n+1) = \frac{2^{n-i}}{2^n-1} : \frac{2^{n+1-i}}{2^{n+1}-1} = \frac{2^{n-i}}{2^n-1} : \frac{2^{n+1}}{2^{n+1-i}} = \frac{2^{n+1}-1}{2(2^n-1)} = \frac{2^{n+1}-1}{2^{n+1}-2} = 1 + \frac{1}{2(2^n-1)} \tag{57}$$

$$w_i^{\mathrm{rg}}(n) : w_i^{\mathrm{rg}}(n+1) = \frac{2^{i-1}}{(2^i-1)G_n} : \frac{2^{i-1}}{(2^i-1)G_{n+1}} = \frac{G_{n+1}}{G_n} = \frac{G_n + \frac{2^n}{2^{n+1}-1}}{G_n} = 1 + \frac{2^n}{(2^{n+1}-1)G_n} \tag{58}$$

$$w_i^{\mathrm{h}}(n) : w_i^{\mathrm{h}}(n+1) = \frac{H_{n+1}}{H_n} = \frac{H_n + \frac{1}{n+1}}{H_n} = 1 + \frac{1}{(n+1)H_n} \tag{59}$$

$$w_i^{\mathrm{rh}}(n):w_i^{\mathrm{rh}}(n+1)=\frac{H_{n+1}^{(-1)}}{H_n^{(-1)}}=\frac{H_n^{(-1)}+\frac{1}{H_n+1}}{H_n^{(-1)}}=1+\frac{1}{H_n^{(-1)}H_{n+1}}=1+\frac{1}{H_n^{(-1)}\left(H_n+\frac{1}{n+1}\right)}=1+\frac{n+1}{[(n+1)H_n+1]H_n^{(-1)}} \quad (60)$$

From Eqs.(55)-(60) follows that for the proportional scheme the weight change $(w_i(n)\div w_i(n+1))$ depends and on the number of co-authors n and the position i. For other schemes it depends only on the number of co-authors.

Results of the comparasion of the weighting schemes are given in Table 1.

From Table 1 we see that with increasing the position i from 1 to $n-1$, the ratio w_i / w_{i+1} for the proportonal scheme increases from $1+\frac{1}{n-1}$ to 2:

$$1+\frac{1}{n-1}=\frac{w_1^{\mathrm{p}}}{w_2^{\mathrm{p}}}<\frac{w_2^{\mathrm{p}}}{w_3^{\mathrm{p}}}<\ldots<\frac{w_{n-1}^{\mathrm{p}}}{w_n^{\mathrm{p}}}=2 \quad (61)$$

Against the proportonal scheme for the relative proportional, relative geometric, harmonic and relative harmonic schemes this ratio decreases with increasing i from 1 to $n-1$:

$$1+\frac{1}{2}=\frac{w_1^{\mathrm{rp}}}{w_2^{\mathrm{rp}}}>1+\frac{1}{3}=\frac{w_2^{\mathrm{rp}}}{w_3^{\mathrm{rp}}}>\ldots>\frac{w_{n-1}^{\mathrm{rp}}}{w_n^{\mathrm{rp}}}=1+\frac{1}{n} \quad (62)$$

$$1+\frac{1}{2}=\frac{w_1^{\mathrm{rg}}}{w_2^{\mathrm{rg}}}>1+\frac{1}{2^3-2}=\frac{w_2^{\mathrm{rg}}}{w_3^{\mathrm{rg}}}>\ldots>\frac{w_{n-1}^{\mathrm{rg}}}{w_n^{\mathrm{rg}}}=1+\frac{1}{2^n-2} \quad (63)$$

$$1+\frac{1}{1}=\frac{w_1^{\mathrm{h}}}{w_2^{\mathrm{h}}}>1+\frac{1}{2}=\frac{w_2^{\mathrm{h}}}{w_3^{\mathrm{h}}}>\ldots>\frac{w_{n-1}^{\mathrm{h}}}{w_n^{\mathrm{h}}}=1+\frac{1}{n} \quad (64)$$

$$1+\frac{1}{(1+1)H_1}=1+\frac{1}{2}=\frac{w_1^{\mathrm{rh}}}{w_2^{\mathrm{rh}}}>1+\frac{1}{(2+1)H_2}=1+\frac{2}{9}=\frac{w_2^{\mathrm{rh}}}{w_3^{\mathrm{rh}}}>\ldots>\frac{w_{n-1}^{\mathrm{rh}}}{w_n^{\mathrm{rh}}}=1+\frac{1}{nH_{n-1}} \quad (64)$$

And for the geometric and fractional schemes the ratio w_i / w_{i+1} is constant for any i (it doesn't depend on the position i and the number of co-authors n), is equal 2 and 1, respectively.

It is obvious that, with increasing the number of co-authors the weights assigned to them decrease, and decreasing rate of change is constant for all positions, in other words it doesn't depend on the position i (see last column of Table 1). And in this case, the proportional scheme is an exception. As can be seen from Table 1 (last column) for this scheme, the decreasing rate of change depends on the position and it decreases from the first to the last position:

$$\frac{w_1^{\mathrm{p}}(n)}{w_1^{\mathrm{p}}(n+1)}=\left(1+\frac{2}{n}\right)\left(1-\frac{1}{n+1}\right)>\frac{w_2^{\mathrm{p}}(n)}{w_2^{\mathrm{p}}(n+1)}=\left(1+\frac{2}{n}\right)\left(1-\frac{1}{n}\right)>\ldots>\frac{w_n^{\mathrm{p}}(n)}{w_n^{\mathrm{p}}(n+1)}=\left(1+\frac{2}{n}\right)\left(1-\frac{1}{2}\right) \quad (66)$$

In other words, with increasing the number co-authors, the weight of the first authors decreases more rapidly than last authors.

Table 1

Comparison of weighting schemes

	w_i / w_{i+1}	$w_i - w_{i+1}$	w_i / w^f	$w_i - w^f$	w_1 / w_n	$w_1 - w_n$	$w_i(n) / w_i(n+1)$
$w_i^{\mathrm{p}} = \frac{2(n-i+1)}{n(n+1)}$	$1+\frac{1}{n-i}$	$\frac{2}{n(n+1)}$	$2-\frac{2i}{n+1}$	$\frac{1}{n}-\frac{2i}{n(n+1)}$	n	$\frac{2(n-1)}{n(n+1)}$	$\left(1+\frac{2}{n}\right)\cdot\left(1-\frac{1}{n+2-i}\right)$
$w_i^{\mathrm{rp}} = \frac{1}{(i+1)P_n}$	$1+\frac{1}{i+1}$	$\frac{1}{(i+1)(i+2)P_n}$	$\frac{n}{(i+1)P_n}$	$\frac{1}{(i+1)P_n}-\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{n-1}{2(n+1)P_n}$	$1+\frac{1}{(n+2)\sum_{j=1}^{n}\frac{1}{j+1}}$
$w_i^{\mathrm{g}} = \frac{2^{n-i}}{2^n-1}$	2	$\frac{2^{n-i-1}}{2^n-1}$	$\frac{n\cdot 2^{n-i}}{2^n-1}$	$\frac{2^{n-i}}{2^n-1}-\frac{1}{n}$	2^{n-1}	$\frac{2^{n-i}-1}{2^n-1}$	$1+\frac{1}{2(2^n-1)}$
$w_i^{\mathrm{rg}} = \frac{2^{i-1}}{(2^i-1)G_n}$	$1+\frac{1}{2^{i+1}-2}$	$\frac{1}{G_n}\cdot\frac{2^{i-1}}{(2^i-1)(2^{i+1}-1)}$	$\frac{2^{i-1}}{2^i-1}\frac{n}{G_n}$	$\frac{1}{G_n}\cdot\frac{2^{i-1}}{2^i-1}-\frac{1}{n}$	$2-\frac{1}{2^{n-1}}$	$\frac{1}{G_n}\cdot\frac{2^{n-1}}{2^n-1}$	$1+\frac{2^n}{(2^{n+1}-1)G_n}$
$w_i^{\mathrm{h}} = \frac{1}{iH_n}$	$1+\frac{1}{i}$	$\frac{1}{i(i+1)H_n}$	$\frac{n}{iH_n}$	$\frac{1}{iH_n}-\frac{1}{n}$	n	$\frac{n-1}{nH_n}$	$1+\frac{1}{(n+1)H_n}$
$w_i^{\mathrm{rh}} = \frac{1}{H_i H_n^{(-1)}}$	$\frac{H_{i+1}}{H_i}=1+\frac{1}{(i+1)H_i}$	$\frac{1}{(i+1)H_i H_{i+1} H_n^{(-1)}}$	$\frac{n}{H_i H_n^{(-1)}}$	$\frac{1}{H_i H_n^{(-1)}}-\frac{1}{n}$	$\frac{H_n}{H_1}=H_n \approx \ln(n)+\gamma$	$\frac{H_n-H_1}{H_1 H_n H_n^{(-1)}}$	$1+\frac{n+1}{[(n+1)H_n+1]H_n^{(-1)}}$
$w_i^{\mathrm{f}} = \frac{1}{n}$	1	0	1	0	1	0	$1+\frac{1}{n}$

It is easy to show that in the geometric scheme, the weight of *i*-th author is always greater than $\frac{1}{2^i}$, regardless of the value of *n*:

$$w_i^{\mathrm{g}} = \frac{2^{n-i}}{2^n - 1} = \frac{2^n}{2^i(2^n - 1)} \xrightarrow[n\to\infty]{} \frac{1}{2^i},\ i = 1,\ldots,n \tag{67}$$

Against the geometric scheme, for the global geometric scheme the weight of co-authors is approaching zero when the number of co-authors increases:

$$w_i^{\mathrm{rg}} = \frac{2^{i-1}}{(2^i - 1)G_n} \xrightarrow[n\to\infty]{} 0,\ i = 1,\ldots,n \tag{68}$$

Below we calculate the weight change of the first from single-authored to *n*-authored paper

$$\frac{w_1^{\mathrm{p}}(1)}{w_1^{\mathrm{p}}(n)} = \frac{n+1}{2} \tag{69}$$

$$\frac{w_1^{\mathrm{rp}}(1)}{w_1^{\mathrm{rp}}(n)} = 2P_n = 2\left(H_n - \frac{n}{n+1}\right) \tag{70}$$

$$\frac{w_1^{\mathrm{g}}(1)}{w_1^{\mathrm{g}}(n)} = \frac{2^n - 1}{2^{n-1}} \tag{71}$$

$$\frac{w_1^{\mathrm{rg}}(1)}{w_1^{\mathrm{rg}}(n)} = G_n = \sum\nolimits_{j=1}^{n} \frac{2^{j-1}}{2^j - 1} \tag{72}$$

$$\frac{w_1^{\mathrm{h}}(1)}{w_1^{\mathrm{h}}(n)} = H_n \tag{73}$$

$$\frac{w_1^{\mathrm{rh}}(1)}{w_1^{\mathrm{rh}}(n)} = H_n^{(-1)} = \sum\nolimits_{j=1}^{n} \frac{1}{\mathrm{H}_j} \tag{74}$$

After analyzing Eqs.(69)-(74) we come to conclusion that the following relation is hold:

$$\frac{w_1^{\mathrm{rg}}(1)}{w_1^{\mathrm{rg}}(n)} > \frac{w_1^{\mathrm{p}}(1)}{w_1^{\mathrm{p}}(n)} > \frac{w_1^{\mathrm{rh}}(1)}{w_1^{\mathrm{rh}}(n)} > \frac{w_1^{\mathrm{rp}}(1)}{w_1^{\mathrm{rp}}(n)} > \frac{w_1^{\mathrm{h}}(1)}{w_1^{\mathrm{h}}(n)} > \frac{w_1^{\mathrm{g}}(1)}{w_1^{\mathrm{g}}(n)} \tag{75}$$

In other words, from single-authored to multi-authored paper weight of the first author is more rapidly decreases than in other schemes.

5. Experiments and analysis

In this section, the proposed indices are compared with other weighted *h*-type indices. 30 researchers were selected from Google Scholar with the *h*-index in range [14, 30] in Data Mining field. These researchers were pointed out from R1 to R30. The number of papers and citations of these researchers, the number of single-authored and first-author, as well as their *h*- and *g*- indices are given in Table 2.

Table 2

Number of papers, citations, *h*- and *g*- indices of researchers

Researchers	Number of papers	Number of citations	*h*-index	*g*-index	Number of single-authored papers	Number of first-authored papers
R1	170	6040	30	76	5	30
R2	159	3887	29	60	9	26
R3	160	2933	29	52	6	11
R4	174	5239	29	70	27	55
R5	159	8290	28	90	24	86
R6	72	6858	28	56	1	4
R7	100	4729	27	65	2	18
R8	146	4675	26	67	12	32
R9	259	2270	25	41	76	111
R10	196	2501	24	45	2	4
R11	143	3920	24	61	3	13
R12	137	2369	24	46	19	49
R13	126	1914	24	41	3	11
R14	96	2417	23	48	4	21
R15	121	1759	23	40	13	28
R16	118	4178	23	64	4	11
R17	33	4584	23	29	3	11
R18	66	5430	22	47	2	9
R19	204	1571	21	34	1	18
R20	97	1414	21	35	3	12
R21	115	1793	20	39	5	24
R22	128	7018	20	83	4	19
R23	142	1744	19	40	3	26
R24	157	1365	19	29	2	35
R25	106	2074	19	44	15	50
R26	67	1622	19	40	1	11
R27	113	888	17	28	1	26
R28	60	5273	17	38	5	10
R29	52	1613	17	40	1	10
R30	118	771	14	23	3	11

h-type indices are shown in Table 3.

Table 3

***h*-type indices of researchers**

Researchers	h	h_f	h_p	h_{rp}	h_g	h_{rg}	h_h	h_{rh}
R1	30	19	18	18	18	18	18	18
R2	29	19	19	18	18	18	17	18
R3	29	17	17	17	16	17	17	17
R4	29	18	18	18	18	19	18	18
R5	28	22	22	22	22	22	22	22
R6	28	13	13	13	11	13	13	13
R7	27	16	16	16	15	16	17	16
R8	26	16	17	17	16	17	17	17
R9	25	22	22	22	22	22	22	22
R10	24	13	9	10	8	11	9	11
R11	24	13	12	12	11	11	11	11
R12	24	16	17	16	18	16	18	16
R13	24	12	12	12	10	12	11	12
R14	23	14	13	14	13	14	13	14
R15	23	17	18	18	18	18	18	18
R16	23	13	14	13	14	13	13	13
R17	23	15	15	14	15	14	14	14
R18	22	13	13	12	13	12	12	12
R19	21	8	8	7	7	7	7	7
R20	21	11	12	12	12	12	12	12
R21	20	10	12	12	12	11	12	11
R22	20	13	13	14	13	13	13	13
R23	19	12	13	13	13	13	13	13
R24	19	10	10	10	10	10	10	10
R25	19	13	14	13	15	13	14	13
R26	19	7	8	8	8	9	8	9
R27	17	8	10	9	8	8	9	9
R28	17	13	14	13	12	13	12	13
R29	17	9	9	8	7	8	7	8
R30	14	7	7	7	6	7	6	7
Average	*22.8*	*13.63*	*13.83*	*13.60*	*13.30*	*13.57*	*13.43*	*13.57*

g-type indices of researchers are given in Table 4.

Table 4
***g*-type indices of researchers**

Researchers	g	g_f	g_p	g_{rp}	g_g	g_{rg}	g_h	g_{rh}
R1	76	43	47	48	41	46	48	46
R2	60	43	40	40	42	41	40	41
R3	52	30	30	29	28	29	29	29
R4	70	44	41	41	40	43	41	43
R5	90	72	77	77	78	75	77	75
R6	56	32	23	24	19	32	24	28
R7	65	29	31	28	27	29	28	29
R8	67	54	53	53	53	54	53	54
R9	41	34	35	35	35	35	35	35
R10	45	22	14	15	11	19	15	18
R11	61	29	28	28	29	29	28	28
R12	46	28	31	31	33	30	31	30
R13	41	19	18	16	15	18	16	17
R14	48	26	26	26	27	26	26	26
R15	40	27	28	27	27	27	27	27
R16	64	33	34	32	32	32	32	32
R17	29	29	29	29	29	29	29	29
R18	47	37	37	35	35	36	35	36
R19	34	13	11	10	8	11	10	11
R20	35	21	19	19	17	19	19	19
R21	39	19	22	23	23	21	23	21
R22	83	57	65	65	65	62	65	62
R23	40	22	26	27	28	25	27	25
R24	29	15	15	15	14	15	15	15
R25	44	25	26	27	28	26	27	26
R26	40	17	16	16	15	16	16	16
R27	28	13	16	16	16	15	16	15
R28	38	36	42	44	38	38	44	38
R29	40	18	17	15	13	17	15	17
R30	23	10	10	9	9	9	9	9
Average	*49.03*	*29.90*	*30.23*	*30.00*	*29.17*	*30.13*	30.00	29.90

Table 5

The CCC coefficient between h-type indices

	h	h_f	h_p	h_{rp}	h_g	h_{rg}	h_h	h_{rh}
h	1.0000	0.2237	0.2055	0.2058	0.1889	0.2118	0.2062	0.2062
h_f	0.2237	1.0000	0.9613	0.9712	0.9351	0.9747	0.9478	0.9742
h_p	0.2055	0.9613	1.0000	0.9859	0.9701	0.9765	0.9776	0.9803
h_{rp}	0.2058	0.9712	0.9859	1.0000	0.9673	0.9926	0.9848	0.9946
h_g	0.1889	0.9351	0.9701	0.9673	1.0000	0.9627	0.9831	0.9620
h_{rg}	0.2118	0.9747	0.9765	0.9926	0.9627	1.0000	0.9800	0.9979
h_h	0.2062	0.9478	0.9776	0.9848	0.9831	0.9800	1.0000	0.9816
h_{rh}	0.2062	0.9742	0.9803	0.9946	0.9620	0.9979	0.9816	1.0000
Average	*0.2069*	*0.8554*	*0.8653*	*0.8717*	*0.8527*	*0.8709*	*0.8659*	*0.8710*
Rank	*8*	*6*	*5*	*1*	*7*	*3*	*4*	*2*

Table 6

Pearson correlation between h-type indices

	h	h_f	h_p	h_{rp}	h_g	h_{rg}	h_h	h_{rh}
h	1.0000	0.7930	0.7153	0.6720	0.7280	0.7381	0.7564	0.7481
h_f	0.7930	1.0000	0.9627	0.9402	0.9500	0.9713	0.9749	0.9745
h_p	0.7153	0.9627	1.0000	0.9820	0.9846	0.9877	0.9790	0.9826
h_{rp}	0.6720	0.9402	0.9820	1.0000	0.9837	0.9726	0.9666	0.9674
h_g	0.7280	0.9500	0.9846	0.9837	1.0000	0.9872	0.9815	0.9843
h_{rg}	0.7381	0.9713	0.9877	0.9726	0.9872	1.0000	0.9927	0.9947
h_h	0.7564	0.9749	0.9790	0.9666	0.9815	0.9927	1.0000	0.9981
h_{rh}	0.7481	0.9745	0.9826	0.9674	0.9843	0.9947	0.9981	1.0000
Average	*0.7358*	*0.9381*	*0.9420*	*0.9264*	*0.9428*	*0.9492*	*0.9499*	*0.9500*
Rank	*8*	*6*	*5*	*7*	*4*	*3*	*2*	*1*

Table 7

The CCC coefficient between g-type indices

	g	g_f	g_p	g_{rp}	g_g	g_{rg}	g_h	g_{rh}
g	1.0000	0.4771	0.4877	0.4788	0.4562	0.4928	0.4788	0.4846
g_f	0.4771	1.0000	0.9710	0.9677	0.9533	0.9919	0.9677	0.9897
g_p	0.4877	0.9710	1.0000	0.9972	0.9883	0.9881	0.9972	0.9930
g_{rp}	0.4788	0.9677	0.9972	1.0000	0.9892	0.9881	**1.0000**	0.9924
g_g	0.4562	0.9533	0.9883	0.9892	1.0000	0.9740	0.9892	0.9817
g_{rg}	0.4928	0.9919	0.9881	0.9881	0.9740	1.0000	0.9881	0.9985
g_h	0.4788	0.9677	0.9972	**1.0000**	0.9892	0.9881	1.0000	0.9928
g_{rh}	0.4846	0.9897	0.9930	0.9924	0.9817	0.9985	0.9924	1.0000
Average	*0.4794*	*0.9026*	*0.9175*	*0.9162*	*0.9046*	*0.9174*	*0.9162*	*0.9189*
Rank	*8*	*7*	*2*	*4*	*6*	*3*	*5*	*1*

Table 8

Pearson correlation between g-type indices

	g	g_f	g_p	g_{rp}	g_g	g_{rg}	g_h	g_{rh}
g	1.0000	0.8666	0.8319	0.8038	0.8204	0.8204	0.8592	0.8521
g_f	0.8666	1.0000	0.9757	0.9620	0.9734	0.9734	0.9937	0.9917
g_p	0.8319	0.9757	1.0000	0.9974	0.9911	0.9888	0.9974	0.9938
g_{rp}	0.8038	0.9620	0.9974	1.0000	0.9908	0.9893	**1.0000**	0.9935
g_g	0.8204	0.9734	0.9911	0.9908	1.0000	0.9782	0.9908	0.9848
g_{rg}	0.8204	0.9734	0.9888	0.9893	0.9782	1.0000	0.9893	0.9987
g_h	0.8592	0.9937	0.9974	**1.0000**	0.9908	0.9893	1.0000	0.9935
g_{rh}	0.8521	0.9917	0.9938	0.9935	0.9848	0.9987	0.9935	1.0000
Average	0.8363	0.9624	0.9680	0.9624	0.9614	0.9626	0.9749	0.9726
Rank	8	5	3	6	7	4	1	2

In Table 3, we observed that there was little difference in the values of all h-type indices (except the h_f) of researchers R1, R4, R5, R6, R9, R10, R15, R20, R23, R24. While the difference between h index and h-type indices is 3 for R9 researchers and is 5 for R15, but for researches R1, R4, R6, R10, etc. this difference is great. From Table 2, we can see that the number of cited papers of R1, R4, R6, and R10 researchers is small either from single-authored.

Case 1: In only 1 (1.79%) out of 72 articles by the researcher R6, he/she is a sole author and in 4 (7.14%) of them he/she is in the first position. The same comparison can be performed for the researcher R16. In only 4 (4.88%) out of 118 articles, he/she is a sole author and in 11 (13.41%) of them, he/she is in the first position.

Case 2: From 174 papers of the researcher **R4**, 27 of them are single-authored papers. We should note that the R4 researcher is in the second rank according to the number of single-authored papers. Single-authored papers involve 19.7% of his/her cited papers. In 55 papers (40.1%), he/she also ranks in the first position. The researcher R5 has 85.5% cited papers, where includes 17.6% sole-authored, 63.2% papers as the first author. He/she is in the first rank according to the number of his/her first author papers. Examining Table 3, we can see that his/her *h*-type index is almost slightly reduced and is constant.

Case 3: Let’s compare R19 and R20 researchers, whose *h*-indices are the same.

According to Table 3, while the difference between the *h*-index and the *h*-type index is 13 and 14 in R19, the difference in R20 is 9. While researcher R19 has 0.7% single-author, and 13.1% the first author papers, but researcher R20 has 4.1% sole-author, and 16.2% the first author papers. The importance of sole-authored and first-author positions in the papers can demonstrate its efficiency not to diminish the *h*-type indices so much.

Case 4: The relative weighted *h*-type indices for the researchers R10 and R14 increased in all cases compared to the corresponding weighted *h*-type indices, and in some cases for R3, R4, R6, R13, R26, R28, and R30 decreased (respectively, in Table 5 we compare the values of the indices h_p and h_{rp}, h_g and h_{rg}, h_h and h_{rh}).

There exist two types of correlation between *h* and *g* type indices: Pearson və concordance correlation (Liu, Tang et al., 2016). The calculated coefficient and obtained results are shown in Tables 5, 6, 7 and 8. The average correlation coefficients of the indices are given in the last row of the tables.

Provided while using weighted *h*-type indices, the question will be which of them should be preferred. Experiments demonstrated that the *h*-index didn’t correlate well with existing indices. Also, the *h*-index has the lowest average agreement coefficient compared to others.

In Table 5, according to the comparison of the average agreement coefficient of the existing and proposed *h*-type indices, the average agreement coefficient of the *h*-index is low than the schemes h_p, h_g and h_h.

In Table 7, according to the results of the average correlation coefficients, the first two places are occupied by h_{rp} and h_{rh}, and the third place is taken by h_{rg} weight schemes. Thus, these indices are strong correlated with all other indices.

The same case is observed in Table 6 based on the results of the Pearson correlation. Thus, the first three places are occupied by the results of weight schemes h_{rh}, h_h, h_{rg}.

In Table 7, the *g*-type has the lowest average agreement coefficients compared to the others. According to the comparison of *g*-type indices with the average agreement coefficient, the average agreement of *g*-index with the weight schemes g_{rh}, g_p, g_{rg}, g_{rp} and g_h is high.

Pearson correlation of each obtained results was evaluated. From Table 6, we can see that:

- *h*-index highly correlated with the h_f (0.7930), and h_{rg} (0.7564) than with the h_g (0.6720).

Appendix

Table 9

The indications for the researcher R17

Rank of paper	Number of citations	Number of co-authors	author position	C_f	Rank	C_p	Rank	C_g	Rank	C_h	Rank	w^{rp}	C_{rp}	Rank	w^{rg}	C_{rg}	Rank	w^{rh}	C_{rh}	Rank
1	1054	4	2	263.5	2	316.2	2	281.1	2	253.0	2	0.260	274.0	2	0.240	253.5	2	0.248	261.4	2
2	804	3	1	268.0	1	402.0	1	459.4	1	438.5	1	0.462	371.5	1	0.447	359.2	1	0.452	363.4	1
3	653	4	2	163.3	3	195.9	3	174.1	3	156.7	3	0.260	169.8	3	0.240	157.1	3	0.248	161.9	3
4	376	3	2	125.3	4	125.3	4	107.4	6	102.5	6	0.308	115.8	4	0.298	112.0	4	0.301	113.2	4
5	333	10	4	33.3	9	42.4	8	20.8	12	28.3	11	0.099	33.0	9	0.092	30.6	9	0.096	32.0	9
6	279	5	1	55.8	7	93.0	6	144.0	4	122.2	4	0.345	96.3	6	0.240	84.9	6	0.319	89.0	6
7	166	2	1	83.0	5	110.7	5	110.7	5	110.7	5	0.600	99.6	5	0.600	99.6	5	0.600	99.6	5
8	110	6	2	18.3	11	26.2	11	27.9	11	22.4	12	0.209	23.0	12	0.176	19.3	12	0.188	20.7	12
9	91	5	3	18.2	12	18.2	13	11.7	21	13.3	16	0.172	15.7	13	0.174	15.8	13	0.174	15.8	13
10	80	1	1	80.0	6	80.0	7	80.0	7	80.0	7	1.000	80.0	7	1.000	80	7	1.000	80.0	7
11	61	4	1	15.3	14	24.4	12	32.5	9	29.3	10	0.390	23.8	11	0.361	22	11	0.371	22.6	11
12	57	4	2	14.3	16	17.1	14	15.2	15	13.7	15	0.260	14.8	14	0.240	13.7	14	0.248	14.1	14
13	54	5	2	10.8	20	14.4	18	13.9	17	11.8	21	0.230	12.4	19	0.203	10.9	20	0.213	11.5	20
14	53	4	2	13.3	18	15.9	15	14.1	16	12.7	18	0.260	13.8	16	0.240	12.7	17	0.248	13.1	16

Contd...

15	47	3	3	15.7	13	7.8	23	6.7	23	8.5	23	0.231	10.9	21	0.255	12	18	0.247	11.6	19
16	45	3	2	15.0	15	15.0	16	12.9	19	12.3	19	0.308	13.9	15	0.298	13.4	15	0.301	13.5	15
17	42	3	3	14.0	17	7.0	24	6.0	24	7.6	24	0.308	9.7	24	0.255	10.7	22	0.247	10.4	23
18	38	5	1	7.6	24	12.7	19	19.6	13	16.6	13	0.345	13.1	18	0.340	11.6	19	0.319	12.1	18
19	35	1	1	35.0	8	35.0	9	35.0	8	35.0	8	1.000	35.0	8	1.000	35.	8	1.000	35.0	8
20	34	3	2	11.3	19	11.3	22	9.7	22	9.3	22	0.308	10.5	23	0.298	10.1	24	0.301	10.2	24
21	30	1	1	30.0	10	30.0	10	30.0	10	30.0	9	1.000	30.0	10	1.000	30	10	1.000	30.0	10
22	29	3	1	9.7	21	14.5	17	16.6	14	15.8	14	0.462	13.4	17	0.447	13	16	0.452	13.1	17
23	24	3	1	8.0	23	12.0	20	13.7	18	13.1	17	0.462	11.1	20	0.447	10.7	23	0.452	10.8	21
24	19	4	2	4.8	25	5.7	25	5.1	25	4.6	25	0.260	4.9	25	0.240	4.6	25	0.248	4.7	25
25	18	2	1	9.0	22	12.0	21	12.0	20	12.0	20	0.600	10.8	22	0.600	10.8	21	0.600	10.8	22
26	15	5	4	3.0	27	2.0	29	1.0	29	1.6	29	0.138	2.1	29	0.162	2.4	29	0.153	2.3	29
27	14	5	2	2.8	29	3.7	26	3.6	26	3.1	26	0.230	3.2	27	0.203	2.8	27	0.213	3.0	27
28	12	4	3	3.0	28	2.4	28	1.6	28	1.9	28	0.195	2.3	28	0.206	2.5	28	0.203	2.4	28
29	11	3	2	3.7	26	3.7	27	3.1	27	3.0	27	0.308	3.4	26	0.298	3.3	26	0.301	3.3	26

According to Eqs.(11), (23) and (25), the weights assigned to the authors are evaluated, and the obtained results are re-ranked, and we get , and . The evaluation of -core, -index, -index, -core can be given in the same way.

- Also h_f-index highly correlates with the h_{rg} (0.9749) than with the h_g (0.9402).
- The h_p-index highly correlated with the h_{rp} (0.9877) and the lowest h_{rg}-index (0.9790).
- Examining the general correlation of the mentioned indices, the highest correlation (0.9877) is between h_p-and h_{rp}- indices, and the lowest correlation (0.6720) is between *h*-index and h_g-index.

According to Table 3, *h*-index was compared with the proposed weighted *h*-index. The comparison of the *h*-index with the available weighted *h*-indices and the proposed modifications of the *h*-index. The example was given based on the indicators of 30 authors. We should note that while the *h*-index of 30 authors varies in the range of [14-30], the h_p-index varies in the range of [7-22], and the h_g- and h_h- indices in the range of [6-22], and the h_{rp}, h_{rg} və h_{rh}-indices in the range of [7-22].

The evaluated values of its *h*-type indices and other indicators were given as an example in Table 9 on the basis of data from the Google Scholar database of the researcher R17, which is given in Table 2 and whose *g*-index is equal to 23. As seen from table, the 29 papers of researcher R17 totally received 4,584 citations. The distribution of citations on papers, the number of authors and the position of this researcher among the authors are reflected (**Appendix**).

6. Conclusion

As noted in the paper, the *h*-index is the main index used to evaluate the researcher. In recent years, different modifications of *h*-index have been proposed, because of the absence of a common scheme for the distribution of citations among the authors in multiple-authorship papers. In this paper, for considering the authors' contribution to the multiple-authorship papers, the "relative first position" concept was introduced and the existing weight schemes were modified based on this principle. Based on the modified weight schemes, the co-author's weights were evaluated according to their position, and the citations were distributed among the authors in proportion to these weights. Then *h*-type indices were evaluated. Pearson and concordance correlation were evaluated between the available *h*-type indices and the proposed weighted *h*-type indices for 30 researchers selected from Google Scholar. While in previous indexes, the first author gained high weight, but this varies in the proposed weight index. This prevents big leaps between weights. Also, the effect of the number of the researcher's single-author and first-author papers on his/her *h*-type indices is shown on the basis of examples. Contrasting with global weight schemes, modified *h*-indices with local weight schemes have a higher level of agreement than other *h*-type indices. In other words, local indices are correlated well with others. This means that if we want to give weight to the citations, the usage of local weight schemes is more appropriate.

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