

AN APPLICATION OF A HYBRID MCDM MODEL FOR PERSONNEL EVALUATION ON THE BASIS OF INFORMATION CULTURE CRITERIA

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Abstract. Personnel evaluation process is aimed at choosing the best alternative to fill the defined vacancy in an organization. It determines the input quality of personnel and thus plays an important role in human resource management. The multi criteria nature and the presence of qualitative factors make it considerably more complex. This paper proposes a hybrid fuzzy MCDM model for personnel evaluation. It combines the fuzzy TOPSIS method with fuzzy worst-case (or entropy) method for linguistic reasoning under group decision making. Fuzzy worst-case and entropy methods are used to get weights of criteria, while fuzzy TOPSIS is utilized to rank the alternatives. The weights obtained from fuzzy worst-case and entropy methods are included in fuzzy TOPSIS computations and the alternatives are evaluated. The fuzzy MCDM for group decision making enables to aggregate subjective assessments of the decision-makers and thus offer an opportunity to perform more robust personnel evaluation procedures. To evaluate the alternatives we have formed an executive group consisting of five decision-makers. For evaluation the group has decided to consider five information culture criteria expressed in linguistic variables. A numerical example demonstrated the possibilities of application of the proposed method.

Keywords: Personnel evaluation; hybrid fuzzy MCDM; fuzzy TOPSIS; fuzzy worst-case method; entropy method; information culture criteria; aggregate rank

1. INTRODUCTION

With the increasing competition in the global market, modern organizations face great challenges. The future survival of companies depends mainly on the contribution of their personnel to companies. Employee or personnel performances such as knowledge, capability, skill and other abilities play an important role in the success of an organization. Therefore, in order to remain a place in the market, it is necessary for companies to put more emphasis on personnel evaluation process (Karsak, 2001; Zhang & Liu, 2011). Personnel evaluation plays an important role in human resource management policy in any company which determines the input quality of personnel. Personnel evaluation is the process of choosing among the alternatives applying for a defined job in the company, the ones who have the qualifications required to perform the job in the best way (Dursun & Karsak, 2010; Balezentis et al., 2012; 2013).

Personnel evaluation is a complex process in the scope of which many factors should be evaluated simultaneously in the decision making process. Evaluation process should provide reliable and valid information about alternatives. There are some conventional techniques used in this process; mainly, completion of application forms, initial interview, employment test and background investigation. The conventional personnel evaluation techniques that are developed on the basis of static job characteristics will no longer suffice. These methods generally come to a conclusion on the basis of the subjective judgment of decision maker, which makes the accuracy of the results highly questionable. Moreover, these methods take into consideration some classical criteria (age, experience etc.) in the decision making process (Dagdeviren, 2010). Various studies have been conducted on personnel evaluation problem to eliminate the drawbacks of conventional personnel evaluation techniques (Chien & Chen, 2008; Karsak, 2001; Zhang & Liu, 2011; Dursun & Karsak, 2010; Wu, 2010). Chien & Chen (2008) reviewed the personnel evaluation studies and found that the important issues including change in organizations, change in work, change in personnel, change in the society, change of laws, and change in marketing have influenced personnel evaluation and recruiting. Personnel recruitment and evaluation directly affect the quality of employees (Chien & Chen, 2008). Hence, various new technologies, like computer-based testing, internet-based testing, telephone-based interviews, video-conference job interviews,



and multimedia simulation tests, allow organizations to test large numbers of applicants at the same time and help saving time and money, and make better personnel evaluation decisions (Oostrom et al. 2013).

The ongoing processes of globalization as well as increasing competition require improving the personnel evaluation process. In recent years, with the rapid development of knowledge economics, the new types of industry have been gradually formed. The core content of this kind of industry includes knowledge, information, creativity, design and symbolic value, etc. In the knowledge economy era, the international competition becomes much fiercer than ever before. To be competent in the international trade stage, a country should not concentrate on the natural resources and the amount of money, but should pay attention on the quantity and quality of the talents who have creative ability and practical ability. All the countries should focus on the competition of the talent cultivation. Information is the major force to promote modern economic growth (Wu, 2010).

With the knowledge-based economy comes an increased awareness of the value of information and knowledge as unique, vital resources and factors of production (Bergeron et al. 2007). In the process of industrialization development together with information technology, the information personnel have become the "first resource". They play a fundamental, strategic and decisive role. So, it is important to build a practical evaluation system and create innovative environment for information personnel training, making information personnel evaluation system (Wu, 2010).

We are living in an era which information is crucial for organizational effectiveness and key to the ability of the organization to respond to change. It is the glue that holds together the structure of organization and is considered a valuable asset in organizational management (Khan & Azmi, 2005; Cameroon & Ettington, 1988; Cameroon & Freeman, 1991; Cameroon & Quinn, 2011). While information culture has been found affects many aspects of organizational behavior. The assumptions, values, and norms that people have about creating, sharing, using information – would have its own effect on organizational behavior and effectiveness (Choo, 2013). The process of organizational management therefore uses information to contribute to the effective decision-making at all level of an organization (Curry & Moore, 2003). Then Sangdee et al. (2009) suggested a number of research propositions that would explore the relationship between information culture and organizational effectiveness.

Recent studies stated that an information culture plays an important role in the success of the modern organizations (Choo, 2013; Steinwachs, 1999). Information culture is an important factor that must be stimulated in all type of modern organization management. Khan & Azmi (2005) explained that information culture is a culture where information is the essence of all activities in organization. Choo et al. (2006) and Bergeron et al. (2007) looked at information culture as the socially shared patterns of behaviors, norms and values that define the significance and use of information in an organization. To profile an organization's information culture, in more recent studies, Choo et al. (2008) adapted six criteria: (1) *Information integrity*; (2) *Information formality*; (3) *Information control*; (4) *Information transparency*; (5) *Information sharing*; and (6) *Information pro-activeness*. In (Choo, 2013) have been represented four information culture types – Result-oriented: it pursues goal achievement, competitive advantage; Rule-following: it pursues control, compliance, accountability; Relationship-based: it encourages communication, participation, commitment; Risk-taking: it encourages innovation, creativity, exploration of new ideas. Each information culture type may be characterized by a set of 5 attributes: the primary goal of information management; information values and norms; information behaviors in terms of information needs, information seeking, and information use. Authors of the work (Alguliev & Mahmudova, 2011) state that information culture of personnel may be evaluated by a set of five skills, namely: 1) *information gathering and perception skill*; 2) *information memorization skill*; 3) *information handling skill*; 4) *information protection and security skill*; 5) *information presentation skill*. So, in this study, these skills are selected as criteria for personnel evaluation which will be explained in Section 4.

It is known that selecting the best alternative among many alternatives is a multi-criteria decision making (MCDM) problem. MCDM is one of the most widely used decision methodologies in science, business, and engineering worlds. MCDM methods aim at improving



the quality of decisions by making the process more explicit, rational, and efficient (Deng et al., 2011; Noor-E-Alama et al., 2011). A typical MCDM problem involves a number of alternatives to be evaluated and a number of criteria to evaluate the alternatives. MCDM methods deal with problems of compromise evaluation of the best solutions from the set of available alternatives according to objectives. In this study, a hybrid model was proposed for the personnel evaluation process using information culture criteria. This paper hence is aimed at combining the fuzzy TOPSIS with other methods for linguistic reasoning under group decision making. Both modified TOPSIS (Patil & Kant, 2014) and "worst-case" (Rotshtein, 2009) (or entropy) methods were utilized within the framework of the proposed model. The "worst-case" method is used to determine the relative weight of the criteria; the modified TOPSIS method is used to rank the alternatives in terms of overall performance with respect to multiple information culture criteria. The combined fuzzy TOPSIS was applied for solving a personnel selection problem.

This paper is organized as follows. We first introduce the literature review in Section 2. In Section 3, hybrid model for personnel evaluation is presented and the stages of the proposed approach and steps are determined in detail. How the proposed model is used on a real world example is explained in Section 4. Finally, we conclude this paper in Section 5.

2. LITERATURE REVIEW

In the literature, there are a number of studies that have been conducted on personnel evaluation. The world around us is difficult to see in one-dimensional way in order to assess what we see. So, we always compare and rank objects of our choice with respect to different criteria. In this context MCDM is a powerful tool widely used in evaluating, selecting or ranking a finite set of decision alternatives characterized by multiple and usually conflicting criteria (Chang et al., 2013; Paksoy et al., 2012). It can help users understand the results of integrated assessments. At present, different researchers have proposed many MCDM methods: the Analytic Hierarchy Process (AHP) (Saaty, 2006), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon, 1981), VIKOR (Su, 2011; Wan et al., 2013). The AHP method allows the decision makers (DMs) to model a complex problem in a hierarchical structure showing the relationships of the goal, objectives (criteria), sub-objectives, and alternatives. The main concept of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive-ideal solution and the longest distance from the negative-ideal solution. Positive-ideal solution is the one that maximizes the benefit criteria and minimizes the cost criteria, while the negative-ideal solution maximizes the cost criteria and minimizes the benefit criteria. This method defines an index called closeness to positive-ideal solution and the farthest from the negative-ideal solution. Finally, this method chooses an alternative with the maximum closeness to the positive-ideal solution (Zhang & Zhang, 2013; Rao, 2013). The AHP method requires pairwise comparison of various alternatives with respect to each of the criteria and pairwise comparison criteria themselves. When the number of alternatives and/or criteria increases then increases the size and number of the comparison matrices. In (Rao, 2013), the AHP method is improved by eliminating the comparison matrices required for alternatives. In the improved AHP method, by normalizing the values of criteria by a systematic way, the rank reversal problem also is removed. To avoid the laborious procedures of generating and processing paired comparison matrices Rotshtein (2009) proposes worst-case method. This method instead of pairwise comparison uses special relations based on comparison with the worst alternative and with the least important criterion. VIKOR is a helpful tool in MCDM, particularly in a situation where the decision maker is not able or does not know to express preference in the beginning of system design. VIKOR ranks alternatives and determines the solution named compromise that is the closest to the ideal. Su (2011) proposed a hybrid fuzzy approach, which assesses each alternative in terms of distance measure calculated by a modified VIKOR method.

The AHP method has been criticized is that decision problems are structured in a hierarchical manner. Some decision-making problems cannot be structured hierarchically because they involve the interaction and dependence of higher level elements on lower elements. Saaty (2008) proposed the use of analytic network process (ANP) to solve the problem of dependence among alternatives and criteria. AHP can effectiveness handle both qualitative and quantitative data, but the conventional AHP still cannot reflect the ambiguity in human thinking style. Therefore, to solve these problems proposed some solutions to fuzzy MCDM problems, such as fuzzy AHP, fuzzy



TOPSIS. They are the key tools used to solve the evaluation and scheduling problems under the fuzzy multi-criteria conditions (Buyukozkan et al., 2012). Güngör et al. (2009) have developed a personnel evaluation system based on fuzzy AHP. Dursun and Karsak (2010) presented a fuzzy MCDM algorithm using the principles of fusion of fuzzy information, 2-tuple linguistic representation model and TOPSIS. Lately, Zhang and Liu (2011) and Guo (2013) used grey relational analysis to solve the personnel evaluation problem under an intuitionistic fuzzy environment. In (Chien & Chen, 2008) have been developed a data mining framework for personnel evaluation to explore the association rules between personnel characteristics and work behaviors, including work performance and retention.

Liang and Wang (1994) proposed an algorithm for personnel evaluation using a fuzzy MCDM method. Numerous fuzzy MCDM methods have been developed and there is no best method for the general fuzzy MCDM problem. Most fuzzy number ranking methods suffer from various drawbacks such as (a) lack of sensitivity when comparing similar fuzzy numbers, (b) counterintuitive outcomes in certain circumstances, and (c) complex computational processes (Deng & Yeh, 2006; Deng et al., 2011). Therefore, in recent years, researchers have attempted to combine different methods to select the best alternative. For supporting the personnel evaluation process in the manufacturing systems Dağdeviren (2010) proposed a hybrid model which employs ANP and modified TOPSIS. Lin (2010) combined ANP with fuzzy data envelopment analysis and proposed an integrated method to solve the personnel evaluation problem. Balezentis et al. (2012) for solving a personnel evaluation problem the new hybrid MULTIMOORA-FG method proposed to cope with group decision making by employing fuzzy weighted averaging operator. Further in (Balezentis & Zeng, 2013), the MULTIMOORA method was extended by employing type-2 fuzzy sets with generalized interval-valued trapezoidal fuzzy numbers. The new fuzzy MULTIMOORA method, as in the case of the crisp MULTIMOORA, consists of the three parts, namely the Ratio System, the Reference Point, and the Full Multiplicative Form, representing different approaches of data aggregation.

The above reviewed personnel evaluation methods operate under the assumption that the evaluation criteria are independent and have same priority levels. However, it is known that this assumption is not always practical. Currently only a few papers address this point. To overcome the problem of criteria dependence, for example, Shyur and Shih (2006) take advantage of ANP instead of AHP to elicit weights of criteria due to their capability of processing the interdependence between criteria. Yu et al. (2013) suggested the prioritized average operator to deal with this situation. Ginevicius (2011) proposed a new method FARE (FACTOR RELATIONSHIP) for determining the criteria weights based on the relationships between all the criteria.

3. THE HYBRID FUZZY MCDM MODEL

For personnel evaluation MCDM has recently been recognized as an efficient method to integrate multiple indices arising from many sources into a single meaningful and overall index. MCDM methods deal with problems of compromise evaluation of the best solutions from the set of available alternatives according to criteria. The MCDM methods consist of the following steps (Klemenis & Askounis, 2010; Jahanshahloo et al., 2006): 1) Formation of the decision making group; 2) Establishing of system evaluation criteria; 3) Generation of the alternatives; 4) Choice of linguistic variables and respective scales for the weights of the criteria and ratings of the alternatives; 5) Selection of criteria weights for each decision maker; 6) Construction of the fuzzy decision matrix; 7) Construction of the normalized fuzzy decision matrix; 8) Construction of the weighted normalized fuzzy decision matrix; 9) Rank the alternatives and accept one alternative as best; 10) If the final solution is not accepted, gather new information and go into the next iteration of multi-criteria optimization.

Let A_i ($i = 1, 2, \dots, n$) be a finite set of n decision alternatives which are to be evaluated by a group of K decision makers DM_k ($k = 1, 2, \dots, K$) with respect to a set of m evaluation criteria C_j ($j = 1, 2, \dots, m$). The evaluation criteria are measurable quantitatively or assessable qualitatively, and are independent of each other. Assessments are to be made by each decision maker DM_k to determine (a) weight vector $\mathbf{W}^k = (w_1^k, w_2^k, \dots, w_m^k)$, and (b) the decision matrix



$X^k = \|x_{ij}^k\|$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, m$; $k = 1, 2, \dots, K$. The weight vector W^k represents the weights of the criteria C_j , which are given by the decision makers DM_k using a cardinal scale. The decision matrix X^k represents the performance ratings assigned to alternative A_i with respect to criteria C_j , which are either objectively measured (for quantitative criteria) or subjectively (for qualitative criteria) assessed by the decision maker DM_k using cardinal values (Chang et al., 2013).

3.1. Ranking the Alternatives: Fuzzy TOPSIS Method

In this study, we utilize this method to evaluate the performance of each alternative. The TOPSIS method (Hwang and Yoon, 1981) is based on the intuitive principle that the best alternatives should have the shortest distance from the positive-ideal alternative and the farthest distance from the negative-ideal alternative. The positive-ideal solution is a hypothetical solution for which all criteria values correspond to the maximum criteria values comprising the satisfying solutions. The negative-ideal solution is a hypothetical solution for which all criteria values correspond to the minimum criteria values comprising the unsatisfying solutions. This method has been widely used in various MCDM models for solving practical decision problems (Chang et al., 2013). This is due to (a) its simplicity and comprehensibility in concept, (b) its computational efficiency, and (c) its ability to measure the relative performance of the decision alternatives in a simple mathematical form.

The TOPSIS method consists of the following steps (Shyur & Shih, 2006; Dagdeviren, 2010; Patil & Kant 2014):

Step 1. *Determine the weighting of evaluation criteria.* To calculate the weights of criteria this study proposes “worst-case” method and the entropy method. These methods are described in Section 3.2.

Step 2. *Construct a decision matrix for the ranking.* The decision matrix X^k can be constructed as follows: $X^k = [x_{ij}^k]$, where x_{ij}^k is the performance rating of alternative A_i with respect to criterion C_j evaluated by k th decision maker DM_k .

Step 3. *Choose the appropriate linguistic variables for the criteria and the alternatives with the respect to criteria.* Due to the uncertainty, the decision maker prefers to give his opinions in linguistic variables. A linguistic variable is a variable whose values are linguistic terms. Each linguistic value can be represented by a fuzzy number which can be assigned to a membership function. Among the various shapes of fuzzy number, triangular fuzzy number (TFN) is the most popular one. It is a fuzzy number represented with three points as follows: $x_{ij}^k = (l_{ij}^k, m_{ij}^k, u_{ij}^k)$, where m_{ij}^k is the most possible assessment value, l_{ij}^k and u_{ij}^k are the lower and upper values respectively for reflecting the fuzziness of the assessment, $l_{ij}^k \leq m_{ij}^k \leq u_{ij}^k$. This representation is interpreted as membership function as follows:

$$\mu_{x_{ij}^k}(x) = \begin{cases} 0, & x < l_{ij}^k \\ \frac{x - l_{ij}^k}{m_{ij}^k - l_{ij}^k}, & l_{ij}^k \leq x \leq m_{ij}^k \\ \frac{u_{ij}^k - x}{u_{ij}^k - m_{ij}^k}, & m_{ij}^k \leq x \leq u_{ij}^k \\ 0, & x > u_{ij}^k \end{cases} \tag{1}$$

Definition. The following are the four operations that can be performed on TFNs. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then,

- (i) **Addition:** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.



(ii) **Subtraction:** $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(iii) **Multiplication:**

$\tilde{A} \times \tilde{B} = (\min\{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}, a_2 b_2, \max\{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\})$.

(iv) **Division:**

$\tilde{A} \div \tilde{B} = (\min\{a_1 \div b_1, a_1 \div b_3, a_3 \div b_1, a_3 \div b_3\}, a_2 / b_2, \max\{a_1 \div b_1, a_1 \div b_3, a_3 \div b_1, a_3 \div b_3\})$

Step 4. Aggregate the weights of the criteria. Many operators have been proposed for aggregating information such as arithmetic averaging, weighted arithmetic averaging, geometric averaging, and weighted geometric averaging (Chang et al., 2013). Arithmetic mean and weighted arithmetic mean are among the most used methods for averaging individual assessments of decision makers.

The aggregated weights $\mathbf{W} = [w_j]$, $w_j = (lw_j, mw_j, uw_j)$, of criteria C_j assessed by the committee of K decision-makers can be calculated as

$$lw_j = \frac{1}{K} \sum_{k=1}^K lw_j^k, \quad mw_j = \frac{1}{K} \sum_{k=1}^K mw_j^k, \quad uw_j = \frac{1}{K} \sum_{k=1}^K uw_j^k \quad j = 1, 2, \dots, m, \quad (2)$$

if the arithmetic mean operator is used. Here $w_j^k = (lw_j^k, mw_j^k, uw_j^k)$ is the weight of the criterion C_j , which is given by the decision maker DM_k , where lw_{ij}^k , mw_{ij}^k and uw_{ij}^k are the lower, middle and upper values respectively for reflecting the fuzziness of the assessment, $0 \leq lw_{ij}^k \leq mw_{ij}^k \leq uw_{ij}^k$.

If the decision makers DM_k have different importance values (i.e. carry different weights) α_k in the decision making process, then the aggregated weights of criteria C_j can be calculated using the weighted arithmetic mean operator

$$lw_j = \frac{\sum_{k=1}^K \alpha_k \cdot lw_j^k}{\sum_{k=1}^K \alpha_k}, \quad mw_j = \frac{\sum_{k=1}^K \alpha_k \cdot mw_j^k}{\sum_{k=1}^K \alpha_k}, \quad uw_j = \frac{\sum_{k=1}^K \alpha_k \cdot uw_j^k}{\sum_{k=1}^K \alpha_k}, \quad j = 1, 2, \dots, m. \quad (3)$$

The aggregated importance weights of criteria can also be calculated using the following equations (Patil & Kant 2014):

$$lw_j = \min_{k=1,2,\dots,K} \{lw_j^k\}, \quad mw_j = \frac{1}{K} \sum_{k=1}^K mw_j^k, \quad uw_j = \max_{k=1,2,\dots,K} \{uw_j^k\}, \quad j = 1, 2, \dots, m. \quad (4)$$

If the decision makers DM_k have different importance values α_k then the aggregated weights of criteria we define as follows:

$$lw_j = \min_{k=1,2,\dots,K} \{\alpha_k \cdot lw_j^k\}, \quad mw_j = \frac{\sum_{k=1}^K \alpha_k \cdot mw_j^k}{\sum_{k=1}^K \alpha_k}, \quad uw_j = \max_{k=1,2,\dots,K} \{\alpha_k \cdot uw_j^k\}, \quad j = 1, 2, \dots, m. \quad (5)$$

Step 5. Calculate aggregate fuzzy ratings for the alternatives. If the fuzzy ratings of all decision makers are described as TFNs $x_{ij}^k = (l_{ij}^k, m_{ij}^k, u_{ij}^k)$, then using the arithmetic mean operator the aggregate fuzzy decision matrix denoted by $\tilde{\mathbf{X}} = [\tilde{x}_{ij}]$, $\tilde{x}_{ij} = (\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{u}_{ij})$ can be defined as follows:

$$\tilde{l}_{ij} = \frac{1}{K} \sum_{k=1}^K l_{ij}^k, \quad \tilde{m}_{ij} = \frac{1}{K} \sum_{k=1}^K m_{ij}^k, \quad \tilde{u}_{ij} = \frac{1}{K} \sum_{k=1}^K u_{ij}^k, \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, m. \quad (6)$$

if the decision makers have same importance values.



In case, when the decision makers DM_k have different importance values α_k ($k=1,2,\dots,K$), then aggregated fuzzy rating $\tilde{x}_{ij} = (\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{u}_{ij})$ of alternative A_i with respect to criterion C_j can be calculated using the weighted arithmetic mean operator:

$$\tilde{l}_{ij} = \frac{\sum_{k=1}^K \alpha_k \cdot l_{ij}^k}{\sum_{k=1}^K \alpha_k}, \quad \tilde{m}_{ij} = \frac{\sum_{k=1}^K \alpha_k \cdot m_{ij}^k}{\sum_{k=1}^K \alpha_k}, \quad \tilde{u}_{ij} = \frac{\sum_{k=1}^K \alpha_k \cdot u_{ij}^k}{\sum_{k=1}^K \alpha_k}, \quad i=1,2,\dots,n \quad j=1,2,\dots,m. \quad (7)$$

In the other way the aggregated fuzzy rating $\tilde{x}_{ij} = (\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{u}_{ij})$ can be defined as follows (Patil & Kant 2014):

$$\tilde{l}_{ij} = \min_{k=1,2,\dots,K} \{l_{ij}^k\}, \quad \tilde{m}_{ij} = \frac{1}{K} \sum_{k=1}^K x_{ij}^k, \quad \tilde{u}_{ij} = \max_{k=1,2,\dots,K} \{u_{ij}^k\}, \quad i=1,2,\dots,n \quad j=1,2,\dots,m. \quad (8)$$

If the decision makers DM_k have different importance values α_k then the aggregated fuzzy rating $\tilde{x}_{ij} = (\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{u}_{ij})$ we define as follows:

$$\tilde{l}_{ij} = \min_{k=1,2,\dots,K} \{\alpha_k \cdot l_{ij}^k\}, \quad \tilde{m}_{ij} = \frac{\sum_{k=1}^K \alpha_k \cdot x_{ij}^k}{\sum_{k=1}^K \alpha_k}, \quad \tilde{u}_{ij} = \max_{k=1,2,\dots,K} \{\alpha_k \cdot u_{ij}^k\}, \quad i=1,2,\dots,n; \quad j=1,2,\dots,m. \quad (9)$$

Step 6. *Normalize the aggregate fuzzy decision matrix.* The normalized aggregate fuzzy decision matrix denoted by $\mathbf{Y} = [y_{ij}]$ we define as follows:

$$y_{ij} = (l_{ij}, m_{ij}, u_{ij}) = \left(\frac{\tilde{l}_{ij}}{\tilde{u}_j^+}, \frac{\tilde{m}_{ij}}{\tilde{u}_j^+}, \frac{\tilde{u}_{ij}}{\tilde{u}_j^+} \right) \text{ and } \tilde{u}_j^+ = \max_{i=1,2,\dots,n} \{\tilde{u}_{ij}\} \text{ (for benefit criteria),} \quad (10)$$

$$y_{ij} = (l_{ij}, m_{ij}, u_{ij}) = \left(\frac{\tilde{l}_{ij}^-}{\tilde{u}_{ij}^-}, \frac{\tilde{l}_j^-}{\tilde{m}_{ij}^-}, \frac{\tilde{l}_j^-}{\tilde{l}_{ij}^-} \right) \text{ and } \tilde{l}_j^- = \min_{i=1,2,\dots,n} \{\tilde{l}_{ij}\} \text{ (for cost criteria).} \quad (11)$$

Step 7. *Construct the weighted normalized fuzzy decision matrix.* The weighted normalized fuzzy decision matrix $\mathbf{Y}^w = [y_{ij}^w]$ is constructed by multiplying the normalized aggregate fuzzy decision matrix $\mathbf{Y} = [y_{ij}]$ with the associated weights $\mathbf{W} = [w_j]$:

$$y_{ij}^w = y_{ij} \otimes w_j, \quad i=1,2,\dots,n; \quad j=1,2,\dots,m. \quad (12)$$

Note that y_{ij}^w is a TFN represented by $y_{ij}^w = (l_{ij}^w, m_{ij}^w, u_{ij}^w)$.

Step 8. *Determine the fuzzy positive-ideal solution and fuzzy negative-ideal solution.* The fuzzy positive-ideal solution A^{w+} and the fuzzy negative-ideal solution A^{w-} are determined based on the weighted normalized ratings as follows:

$$A^{w+} = (a_1^{w+}, a_2^{w+}, \dots, a_m^{w+}) \text{ where } a_j^{w+} = (u_j^{w+}, u_j^{w+}, u_j^{w+}) \text{ and } u_j^{w+} = \max_{i=1,2,\dots,n} \{u_{ij}^w\}, \quad (13)$$

$$A^{w-} = (a_1^{w-}, a_2^{w-}, \dots, a_m^{w-}) \text{ where } a_j^{w-} = (l_j^{w-}, l_j^{w-}, l_j^{w-}) \text{ and } l_j^{w-} = \min_{i=1,2,\dots,n} \{l_{ij}^w\}, \quad (14)$$

Step 9. *Calculate the distance of each alternative from the fuzzy positive-ideal solution and fuzzy negative-ideal solution.* We compute the separation distance of each alternative $A_i = (y_{i1}^w, y_{i2}^w, \dots, y_{im}^w)$ from the fuzzy positive-ideal solution $A^{w+} = (a_1^{w+}, a_2^{w+}, \dots, a_m^{w+})$ based on Euclidean distance using the distance measurement between two fuzzy numbers

$$D_i^+ = \sqrt{\sum_{j=1}^m (\text{dist}(y_{ij}^w, a_j^{w+}))^2}. \quad (15)$$



Similarly, the separation distance of each alternative $A_i = (y_{i1}^w, y_{i2}^w, \dots, y_{im}^w)$ from the fuzzy negative-ideal solution $A^{w-} = (a_1^{w-}, a_2^{w-}, \dots, a_m^{w-})$ can be calculated as:

$$D_i^- = \sqrt{\sum_{j=1}^m (\text{dist}(y_{ij}^w, a_j^{w-}))^2} \tag{16}$$

The distances $\text{dist}(y_{ij}^w, a_j^{w+})$ and $\text{dist}(y_{ij}^w, a_j^{w-})$ between two TFNs $y_{ij}^w = (l_{ij}^w, m_{ij}^w, u_{ij}^w)$ and $a_j^{w+} = (u_j^{w+}, u_j^{w+}, u_j^{w+})$, and between two TFNs $y_{ij}^w = (l_{ij}^w, m_{ij}^w, u_{ij}^w)$ and $a_j^{w-} = (l_j^{w-}, l_j^{w-}, l_j^{w-})$ are calculated, respectively, as:

$$\text{dist}(y_{ij}^w, a_j^{w+}) = \sqrt{\frac{1}{3} [(l_{ij}^w - u_j^{w+})^2 + (m_{ij}^w - u_j^{w+})^2 + (u_{ij}^w - u_j^{w+})^2]} \tag{17}$$

$$\text{dist}(y_{ij}^w, a_j^{w-}) = \sqrt{\frac{1}{3} [(l_{ij}^w - l_j^{w-})^2 + (m_{ij}^w - l_j^{w-})^2 + (u_{ij}^w - l_j^{w-})^2]} \tag{18}$$

Step 10. Calculate the closeness index (CI_{*i*}) of each alternative. The closeness index CI_{*i*} represents distances to the fuzzy positive-ideal solution A^+ and the fuzzy negative-ideal solution A^- simultaneously. The closeness index CI_{*i*} of each alternative A_i is evaluated as follows:

$$CI_i = \frac{D_i^-}{D_i^- + D_i^+}, \quad i = 1, 2, \dots, n. \tag{19}$$

Since $D_i^+ \geq 0$ and $D_i^- \geq 0$, then, clearly, the value of CI_{*i*} lies between 0 and 1. The larger the index value of CI_{*i*}, the better performance of the alternatives.

Step 11. Rank the alternatives. Rank the alternatives A_i in accordance with the values of CI_{*i*} in descending order and select the alternative with the highest CI_{*i*} value.

In the above process the normalized decision matrix $Y^w = [y_{ij}^w]$ is weighted by multiplying each column of the matrix $Y = [y_{ij}]$ by its associated weights $W = [w_j]$ of criteria. And the overall performance of an alternative is evaluated by its Euclidean distance to A^{w+} and A^{w-} . However, as reported in (Shyur & Shih, 2006), this distance is interrelated with the criteria weights and should be incorporated in the distance measurement. This because all alternatives are compared with A^{w+} and A^{w-} , rather than directly themselves. Therefore, we present weighted Euclidean distance instead of creating a weighted decision matrix. In this case, we define the fuzzy positive-ideal solution (A^+) and the fuzzy-negative ideal solution (A^-), which are depend on the normalized fuzzy decision matrix $Y = [y_{ij}]$:

$$A^+ = (a_1^+, a_2^+, \dots, a_m^+) \text{ where } a_j^+ = (u_j^+, u_j^+, u_j^+) \text{ and } u_j^+ = \max_{i=1,2,\dots,n} \{u_{ij}\}, \tag{20}$$

$$A^- = (a_1^-, a_2^-, \dots, a_m^-) \text{ where } a_j^- = (l_j^-, l_j^-, l_j^-) \text{ and } l_j^- = \min_{i=1,2,\dots,n} \{l_{ij}\}. \tag{21}$$

Then the weighted Euclidean distances, between $A_i = (y_{i1}, y_{i2}, \dots, y_{im})$ and $A^+ = (a_1^+, a_2^+, \dots, a_m^+)$, $A_i = (y_{i1}, y_{i2}, \dots, y_{im})$ and $A^- = (a_1^-, a_2^-, \dots, a_m^-)$, are computed, respectively, as

$$D_i^+ = \sqrt{\sum_{j=1}^m \tilde{w}_j (\text{dist}(y_{ij}, a_j^+))^2}, \tag{22}$$

$$D_i^- = \sqrt{\sum_{j=1}^m \tilde{w}_j (\text{dist}(y_{ij}, a_j^-))^2}, \tag{23}$$



where

$$\text{dist}(y_{ij}, \alpha_j^+) = \sqrt{\frac{1}{3} [(l_{ij} - u_j^+)^2 + (m_{ij} - u_j^+)^2 + (u_{ij} - u_j^+)^2]}, \tag{24}$$

$$\text{dist}(y_{ij}, \alpha_j^-) = \sqrt{\frac{1}{3} [(l_{ij} - l_j^-)^2 + (m_{ij} - l_j^-)^2 + (u_{ij} - l_j^-)^2]}, \tag{25}$$

In Eqs.(22) and (23), \tilde{w}_j is the defuzzified value of w_j . Defuzzification is the process to convert a fuzzy number into a crisp (non-fuzzy) value. The center-of-area (COA) method is the most popular and commonly used method to defuzzify a TFN. The defuzzification value using this method is obtained by

$$\tilde{w}_j = \frac{lw_j + mw_j + uw_j}{3}. \tag{26}$$

In the above process to derive group preferences provided by multiple decision makers was utilized fuzzy aggregation process, provided in Step 5. Instead of the Step 5 the aggregation can be derived in other way. For this purpose, for each decision maker DM_k we calculate a set of weighted Euclidean distances D_{ik}^+ and D_{ik}^- by Eqs.(22) and (23). Both distances from each decision maker can be aggregated as the distances of the group by taking the arithmetic mean operator:

$$\bar{D}_i^+ = \frac{1}{K} \sum_{k=1}^K D_{ik}^+, \quad \bar{D}_i^- = \frac{1}{K} \sum_{k=1}^K D_{ik}^-, \quad i = 1, 2, \dots, n, \tag{27}$$

if the decision makers have same importance values or by taking the weighted arithmetic mean operator:

$$\bar{D}_i^+ = \frac{\sum_{k=1}^K \alpha_k \cdot D_{ik}^+}{\sum_{k=1}^K \alpha_k}, \quad \bar{D}_i^- = \frac{\sum_{k=1}^K \alpha_k \cdot D_{ik}^-}{\sum_{k=1}^K \alpha_k} \quad i = 1, 2, \dots, n, \tag{28}$$

if the decision makers have different importance values.

Thus, referred to Eq.(19), we can also define the group's aggregated closeness index in the following form:

$$\bar{CI}_i = \frac{\bar{D}_i^-}{\bar{D}_i^- + \bar{D}_i^+}, \quad i = 1, 2, \dots, n. \tag{29}$$

3.2. Calculate the Weights of Criteria

The weights of the criteria we define by worst-case and entropy methods.

Worst-Case Method (WCM). The idea of the worst-case method (Rothstein, 2009) is borrowed from structural system analysis, where the reliability of a system is distributed among its elements according to their ranks. The higher is the rank, the greater is the reliability part. Unlike previous methods (Hwang & Yoon, 1981; Saaty, 2006; Patil & Kant, 2014; Lin, 2010), where for determination of the weights of criteria has been used technique of paired comparison, this method compares criteria only with the one that is the least important among them.

Let w_j^k be the weight of the criterion C_j given by the decision maker DM_k that reflects its importance. Let us suppose that the larger is the weight w_j^k of the criterion C_j , the higher rank is its rank R_j^k . This is formalized by the relation

$$\frac{w_1^k}{R_1^k} = \frac{w_2^k}{R_2^k} = \dots = \frac{w_g^k}{R_g^k} = \dots = \frac{w_m^k}{R_m^k}. \tag{30}$$

Let w_q^k and R_q^k represent the weight and the rank of the least important criterion, respectively, evaluated by the decision maker DM_k . Thus, from Eq.(30) we obtain the following



expression for the weights of all criteria relative to the least important criterion, evaluated by the decision maker DM_k :

$$w_1^k = R_1^k \frac{w_q^k}{R_q^k}, w_2^k = R_2^k \frac{w_q^k}{R_q^k}, \dots, w_m^k = R_m^k \frac{w_q^k}{R_q^k}, k=1,2,\dots,K. \quad (31)$$

Let us require the following condition be hold

$$w_1^k + w_2^k + \dots + w_q^k + \dots + w_m^k = 1, k=1,2,\dots,K. \quad (32)$$

Substituting Eq.(31) into Eq.(32), we obtain the weight of the least important criterion

$$w_q^k = \frac{1}{\frac{R_1^k}{R_q^k} + \frac{R_2^k}{R_q^k} + \dots + \frac{R_m^k}{R_q^k}} = \frac{1}{\sum_{j=1}^m \frac{R_j^k}{R_q^k}}, k=1,2,\dots,K. \quad (33)$$

Eqs.(31) and (33) allow one to calculate the criteria weights using ratios of the ranks of all criteria C_j to the rank of the least important criterion C_q . Note that comparison with the least

important case guarantees that the condition $\frac{R_j^k}{R_q^k} \geq 1$ holds for all $j=1,2,\dots,m$ and $k=1,2,\dots,K$.

In Eq.(33), the ratios $\frac{R_j^k}{R_q^k}$ of criteria ranks are estimated using the Saaty's 1-9 scales (Saaty,

2006; Saaty, 2008). The 1-9 scales are illustrated in Table 1 (see Section 4).

From the Eqs.(31) and (33), using the arithmetic and weighted arithmetic operators, we obtain the following aggregated weights of criteria, respectively:

$$w_j = \frac{1}{K} \sum_{k=1}^K w_j^k, \quad (34)$$

$$w_j = \frac{\sum_{k=1}^K \alpha_k \cdot w_j^k}{\sum_{k=1}^K \alpha_k}. \quad (35)$$

Entropy Method. Another objective way to calculate the weights of criteria is to use Shannon entropy based on the proportion of the j th column of the decision matrix $\mathbf{X}^k = [x_{ij}^k]$:

$$P_{ij}^k = \frac{x_{ij}^k}{\sum_{l=1}^n x_{lj}^k}, i=1,2,\dots,n; j=1,2,\dots,m; k=1,2,\dots,K. \quad (36)$$

For the j th column the entropy is computed as:

$$\varphi_j^k = -\frac{1}{\log n} \sum_{i=1}^n \tilde{P}_{ij}^k \log(\tilde{P}_{ij}^k), \quad (37)$$

where \tilde{P}_{ij}^k is the defuzzified value of the $P_{ij}^k = (lP_{ij}^k, mP_{ij}^k, uP_{ij}^k)$, which is defined analogously to the Eq.(26):

$$\tilde{P}_{ij}^k = \frac{lP_{ij}^k + mP_{ij}^k + uP_{ij}^k}{3}, \quad (38)$$

The quantity φ_j^k essentially provides a measure of closeness of the different proportions. The smaller value of φ_j^k , the larger the variation among the proportions for classifying the rows. So, we can select the weights as:



$$w_j^k = \frac{(1-\varphi_j^k)}{\sum_{s=1}^m (1-\varphi_s^k)}, \quad j=1,2,\dots,m; \quad k=1,2,\dots,K. \quad (39)$$

Analogously to worst-case method and in this case the aggregated weights of criteria can be also obtained using the Eqs.(34) and (35).

4. NUMERICAL EXAMPLE: AN EMPIRICAL APPLICATION

In this section, we conduct a numerical example to illustrate the proposed model for decision making problems with fuzzy data. A personnel evaluation problem will illustrate the group decision-making procedure according to the proposed model. We have formed an executive committee consisting of five decision makers DM_1, DM_2, DM_3, DM_4 and DM_5 to choose the best alternative from another five participants (A_1, A_2, A_3, A_4 and A_5) to fill the vacancy. The committee has decided to consider the following five evaluation criteria: (1) *Information-gathering and perception culture* (C_1): Information Gathering is not limited to passive gather of only provided information by a person. It's a process that starts with comprehension of information demand necessary for solution of any problem and combines skills such as information possessing on existing information resources and their structure, knowledge of information search algorithm in different information sources, determination of list of terms and key words in searched subject, use of traditional and electronic information retrieval engines, comprehension of obtained information regardless of presentation method or type (text, audio, video, scheme, graph etc.). (2) *Information memorization culture* (C_2): Information memorization is one of the important components of information culture of an individual. Memorization process is connected to characteristics such as human memory and attention. Memory has a significant role and importance for succeeding in life. According to scientists, memory is the ability of a person to store obtained knowledge and experience and use them during his life and activities. (3) *Information handling culture* (C_3): New skills are required from people in the period of rapid changes occurring in society, and fast growth of information knowledge. A person must be able of analyzing, evaluation processing information related to his activity field and creating new information. Otherwise, it will be difficult to use obtained informational and knowledge during the decision making process regarding a certain issue. As along with abundance of information, availability of needless, uncertain information causes confusion. Thus, ability to evaluate, select and analyze information is significant. (4) *Information protection and security culture* (C_4): New value of information and its conversion into strategic resource brings its protection and provision of its safety to the foreground in modern period. Solution of information security problem depends not only on technical methods and devices, but also culture of people. Currently, ability to implement the processes of collection, storage, processing and transfer of information using computer, Internet and other technical devices, increases the risk of interception of confidential information by others. Information confidentiality problem is one of the main conditions of provision of information security. People working with information resources must comprehend their responsibility to protect the confidentiality of information belonging to different citizens or organizations. (5) *Information presentation culture* (C_5): A person exhibits possession of rich knowledge, valuable information, when he can present carried knowledge and information to society. Information presentation is usually carried out through oral speech and writing, each of which has its own characteristics. The person mastering these characteristics well can present information on a perfect level. For example, information presentation skills of a person can be highly evaluation, when information is clear to those being presented to; i.e. it is composed of simple and substantial sentences, excessive use of special terms is avoided, carried ideas are consecutive and complete each other. At the same time, field related to the presented information must be well researched, analyzed and referred when necessary.

To assess the importance of the criteria and to evaluate the suitability of the alternatives under each of the criteria the decision-makers use the linguistic variables reported in Tables 1 and 2.



Table 1. Scale of the relative importance of criteria

Intensity of importance, R_j / R_q	The relative importance of the criteria C_j and C_q	Explanation
1	Equal importance	The criterion C_j is equally important as the criterion C_q
2	Weak importance	Intermediate between 1 and 3
3	Moderate importance	The criterion C_j is moderately more important than the criterion C_q .
4	Moderate more importance	Intermediate between 3 and 5
5	Strong importance	The criterion C_j is strongly more important than the criterion C_q .
6	Strong more importance	Intermediate between 5 and 7
7	Very strong importance	The criterion C_j is very strongly more important than the criterion C_q
8	Very strong more importance	Intermediate between 7 and 9
9	Extreme importance	The criterion C_j is extremely more important than the criterion C_q .

Table 2. Linguistic scales for the rating of each alternative

Intensity of performance	Definition	Membership function
1	Very good	(8,9,10)
2	Good	(6,7,8)
3	Fair	(4,5,6)
4	Poor	(2,3,4)
5	Very poor	(1,1,2)

Decision makers have presented their evaluation information for five alternatives in Tables 3-7.

Table 3. Individual fuzzy decision matrix of DM_1

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(6, 7, 8)	(4, 5, 6)	(1, 1, 2)	(2, 3, 4)	(6, 7, 8)
A_2	(4, 5, 6)	(8, 9, 10)	(4, 5, 6)	(1, 1, 2)	(2, 3, 4)
A_3	(8, 9, 10)	(6, 7, 8)	(2, 3, 4)	(4, 5, 6)	(1, 1, 2)
A_4	(2, 3, 4)	(1, 1, 2)	(6, 7, 8)	(4, 5, 6)	(4, 5, 6)
A_5	(4, 5, 6)	(2, 3, 4)	(1, 1, 2)	(6, 7, 8)	(8, 9, 10)

Table 4. Individual fuzzy decision matrix of DM_2

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(4, 5, 6)	(2, 3, 4)	(1, 1, 2)	(2, 3, 4)	(4, 5, 6)
A_2	(2, 3, 4)	(6, 7, 8)	(2, 3, 4)	(1, 1, 2)	(1, 1, 2)
A_3	(8, 9, 10)	(4, 5, 6)	(1, 1, 2)	(4, 5, 6)	(2, 3, 4)
A_4	(1, 1, 2)	(2, 3, 4)	(4, 5, 6)	(6, 7, 8)	(6, 7, 8)
A_5	(6, 7, 8)	(1, 1, 2)	(2, 3, 4)	(8, 9, 10)	(4, 5, 6)



Table 5. Individual fuzzy decision matrix of DM_3

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(8, 9, 10)	(4, 5, 6)	(2, 3, 4)	(4, 5, 6)	(8, 9, 10)
A_2	(6, 7, 8)	(1, 1, 2)	(4, 5, 6)	(2, 3, 4)	(4, 5, 6)
A_3	(2, 3, 4)	(6, 7, 8)	(4, 5, 6)	(1, 1, 2)	(8, 9, 10)
A_4	(4, 5, 6)	(8, 9, 10)	(1, 1, 2)	(6, 7, 8)	(1, 1, 2)
A_5	(2, 3, 4)	(4, 5, 6)	(6, 7, 8)	(8, 9, 10)	(2, 3, 4)

Table 6. Individual fuzzy decision matrix of DM_4

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(4, 5, 6)	(1, 1, 2)	(4, 5, 6)	(6, 7, 8)	(6, 7, 8)
A_2	(6, 7, 8)	(2, 3, 4)	(6, 7, 8)	(4, 5, 6)	(2, 3, 4)
A_3	(8, 9, 10)	(6, 7, 8)	(4, 5, 6)	(4, 5, 6)	(4, 5, 6)
A_4	(1, 1, 2)	(8, 9, 10)	(2, 3, 4)	(4, 5, 6)	(6, 7, 8)
A_5	(2, 3, 4)	(4, 5, 6)	(2, 3, 4)	(6, 7, 8)	(8, 9, 10)

Table 7. Individual fuzzy decision matrix of DM_5

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(6, 7, 8)	(1, 1, 2)	(6, 7, 8)	(2, 3, 4)	(6, 7, 8)
A_2	(2, 3, 4)	(6, 7, 8)	(4, 5, 6)	(1, 1, 2)	(8, 9, 10)
A_3	(8, 9, 10)	(4, 5, 6)	(1, 1, 2)	(6, 7, 8)	(1, 1, 2)
A_4	(1, 1, 2)	(2, 3, 4)	(6, 7, 8)	(4, 5, 6)	(6, 7, 8)
A_5	(6, 7, 8)	(4, 5, 6)	(2, 3, 4)	(6, 7, 8)	(1, 1, 2)

Using the aggregation strategies, the arithmetic mean (Eq.(6)), weighted arithmetic mean (Eq.(7)), Patil & Kant (Eq.(8)) and weighted Patil & Kant (Eq.(9)) operators, from the Tables 3-7 we obtain the following group decision matrices (Tables 8-11):

Table 8. Group fuzzy decision matrix \tilde{X}_{AM} obtained by AM* operator

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(5.60,6.60,7.60)	(2.40,3.00,4.00)	(2.80,3.40,4.40)	(3.20,4.20,5.20)	(6.00,7.00,8.00)
A_2	(4.00,5.00,6.00)	(4.60,5.40,6.40)	(4.00,5.00,6.00)	(1.80,2.20,3.20)	(3.40,4.20,5.20)
A_3	(6.80,7.80,8.80)	(5.20,6.20,7.20)	(2.40,3.00,4.00)	(3.80,4.60,5.60)	(3.20,3.80,4.80)
A_4	(1.80,2.20,3.20)	(4.20,5.00,6.00)	(3.80,4.60,5.60)	(4.80,5.80,6.80)	(4.60,5.40,6.40)
A_5	(4.00,5.00,6.00)	(3.00,3.80,4.80)	(2.60,3.40,4.40)	(6.80,7.80,8.80)	(4.60,5.40,6.40)

Table 9. Group fuzzy decision matrix \tilde{X}_{WAM} obtained by WAM* operator

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(5.60,6.60,7.60)	(2.47,3.16,4.16)	(2.53,3.06,4.06)	(2.92,3.92,4.92)	(5.84, 6.84, 7.84)
A_2	(3.70,4.70,5.70)	(4.76,5.54,6.54)	(3.64,4.64,5.64)	(1.58,1.92,2.92)	(3.28, 3.98, 4.98)
A_3	(6.68,7.68,8.68)	(5.02,6.02,7.02)	(2.19,2.70,3.70)	(3.72,4.50,5.50)	(3.20, 3.84, 4.84)
A_4	(1.83,2.22,3.22)	(3.87,4.70,5.70)	(3.82,4.60,5.60)	(5.04,6.04,7.04)	(4.56, 5.34, 6.34)
A_5	(4.30,5.30,6.30)	(2.76,3.46,4.46)	(2.71,3.54,4.54)	(7.04,8.04,9.04)	(4.15, 4.96, 5.96)



Table 10. Group fuzzy decision matrix \tilde{X}_{PK} obtained by PK* operator

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(4.00, 6.60, 10.00)	(1.00, 3.00, 6.00)	(1.00, 3.40, 8.00)	(2.00, 4.20, 8.00)	(4.00, 7.00, 10.00)
A_2	(2.00, 5.00, 8.00)	(1.00, 5.40, 10.00)	(2.00, 5.00, 8.00)	(1.00, 2.20, 6.00)	(1.00, 4.20, 10.00)
A_3	(2.00, 7.80, 10.00)	(4.00, 6.20, 8.00)	(1.00, 3.00, 6.00)	(1.00, 4.60, 8.00)	(1.00, 3.80, 10.00)
A_4	(1.00, 2.20, 6.00)	(1.00, 5.00, 10.00)	(1.00, 4.60, 8.00)	(4.00, 5.80, 8.00)	(1.00, 5.40, 8.00)
A_5	(2.00, 5.00, 8.00)	(1.00, 3.80, 4.80)	(1.00, 3.40, 8.00)	(6.00, 7.80, 10.00)	(1.00, 5.40, 10.00)

Table 11. Group fuzzy decision matrix \tilde{X}_{WPK} obtained by WPK* operator

Alternatives	Criteria				
	C_1	C_2	C_3	C_4	C_5
A_1	(0.48, 6.60, 2.20)	(0.12, 3.16, 1.32)	(0.17, 3.06, 1.52)	(0.34, 3.92, 1.32)	(0.72, 6.84, 2.20)
A_2	(0.38, 4.70, 1.76)	(0.22, 5.54, 2.40)	(0.60, 4.64, 1.32)	(0.17, 1.92, 0.88)	(0.24, 3.98, 1.90)
A_3	(0.44, 7.68, 3.00)	(0.72, 6.02, 1.80)	(0.19, 2.70, 1.32)	(0.22, 4.50, 1.80)	(0.17, 3.84, 2.20)
A_4	(0.12, 2.22, 1.32)	(0.17, 4.70, 2.20)	(0.22, 4.60, 1.80)	(0.48, 6.04, 2.40)	(0.22, 5.34, 2.40)
A_5	(0.24, 5.30, 2.40)	(0.30, 3.46, 1.32)	(0.17, 3.54, 1.76)	(0.72, 8.04, 3.00)	(0.19, 4.96, 1.80)

Note: AM – Arithmetic Mean, WAM – Weighted Arithmetic Mean, PK – Patil & Kant, WPK – weighted Patil & Kant

To calculate the weights of criteria we first apply the entropy method. From Tables 3-7, using the Eqs.(26), (34)-(39) we obtain the weights of criteria for each decision maker and the average weights of criteria (Table 12). Last two columns of Table 12 are calculated using the Eqs.(34) and (35), respectively, that correspond to cases where DMs have equal and different importance values. In this study, the DMs have the following importance values: $\alpha_1 = 0.17$, $\alpha_2 = 0.30$, $\alpha_3 = 0.22$, $\alpha_4 = 0.12$, and $\alpha_5 = 0.19$.

Table 12. The weights of criteria obtained by entropy method

The weights of criteria for each decision maker (w_j^k)					The average weights of criteria (w_j)	
DM ₁	DM ₂	DM ₃	DM ₄	DM ₅	DMs have equal importance values	DMs have different importance values
$w_1^1 = 0.1565$	$w_1^2 = 0.1750$	$w_1^3 = 0.1825$	$w_1^4 = 0.1945$	$w_1^5 = 0.1716$	$w_1 = 0.1760$	$w_1^\alpha = 0.1752$
$w_2^1 = 0.1934$	$w_2^2 = 0.2060$	$w_2^3 = 0.1912$	$w_2^4 = 0.2275$	$w_2^5 = 0.2124$	$w_2 = 0.2061$	$w_2^\alpha = 0.2044$
$w_3^1 = 0.2468$	$w_3^2 = 0.2527$	$w_3^3 = 0.2222$	$w_3^4 = 0.2205$	$w_3^5 = 0.2032$	$w_3 = 0.2291$	$w_3^\alpha = 0.2317$
$w_4^1 = 0.2098$	$w_4^2 = 0.1748$	$w_4^3 = 0.2055$	$w_4^4 = 0.1800$	$w_4^5 = 0.2031$	$w_4 = 0.1946$	$w_4^\alpha = 0.1935$
$w_5^1 = 0.1935$	$w_5^2 = 0.1916$	$w_5^3 = 0.1985$	$w_5^4 = 0.1774$	$w_5^5 = 0.2096$	$w_5 = 0.1934$	$w_5^\alpha = 0.1952$

For calculation of the criteria weights by the worst-case method the decision makers independently identified the least important criterion and accordingly its rank. Then, using the Saaty's scales, they determined rank other criteria relative to the least important criterion. Table 13 represents the ranks of criteria assigned by each decision maker.

Table 13. The ranks of criteria assigned by each decision maker

Criteria	The relative importance of the criteria defined by each decision maker				
	DM ₁	DM ₂	DM ₃	DM ₄	DM ₅
	R_j / R_5	R_j / R_3	R_j / R_2	R_j / R_4	R_j / R_1
C_j	2	8	5	3	1
C_2	6	4	1	2	3
C_3	7	1	3	6	7
C_4	3	5	8	1	2
C_5	1	7	6	8	4



As seen from the Table 13 for decision makers DM_1, DM_2, DM_3, DM_4 and DM_5 the least important criteria are C_5, C_3, C_2, C_4 and C_1 , respectively. From Table 13, by applying the Eqs.(33) and (31), we obtain the following weights of criteria (Table 14).

Table 14. The weights of criteria obtained by worst-case method

The weights of criteria for each decision maker (w_j^k)					The average weights of criteria (w_j)	
DM_1	DM_2	DM_3	DM_4	DM_5	DMs have equal importance values	DMs have different importance values
$w_1^1 = 0.1053$	$w_1^2 = 0.3200$	$w_1^3 = 0.2174$	$w_1^4 = 0.1500$	$w_1^5 = 0.0588$	$w_1 = 0.1703$	$w_1^\alpha = 0.1909$
$w_2^1 = 0.3158$	$w_2^2 = 0.1600$	$w_2^3 = 0.0435$	$w_2^4 = 0.1000$	$w_2^5 = 0.1765$	$w_2 = 0.1591$	$w_2^\alpha = 0.1568$
$w_3^1 = 0.3684$	$w_3^2 = 0.0400$	$w_3^3 = 0.1304$	$w_3^4 = 0.3000$	$w_3^5 = 0.4118$	$w_3 = 0.2501$	$w_3^\alpha = 0.2176$
$w_4^1 = 0.1579$	$w_4^2 = 0.2000$	$w_4^3 = 0.3478$	$w_4^4 = 0.0500$	$w_4^5 = 0.1176$	$w_4 = 0.1747$	$w_4^\alpha = 0.1917$
$w_5^1 = 0.0526$	$w_5^2 = 0.2800$	$w_5^3 = 0.2609$	$w_5^4 = 0.4000$	$w_5^5 = 0.2353$	$w_5 = 0.2458$	$w_5^\alpha = 0.2430$

The normalized aggregate fuzzy decision matrices which obtained from the Tables 8-11 by utilizing the Eq.(10), are shown in Tables 15-18.

Table 15. The normalized aggregate fuzzy decision matrix Y_{AM} obtained from the matrix \tilde{X}_{AM}

Criteria	C_1	C_2	C_3	C_4	C_5
A_1	(0.6364,0.7500,0.8636)	(0.3333,0.4167,0.5556)	(0.4667,0.5667,0.7333)	(0.3636,0.4773,0.5909)	(0.7500,0.8750,1.0000)
A_2	(0.4545,0.5682,0.6818)	(0.6389,0.7500,0.8889)	(0.6667,0.8333,1.0000)	(0.2045,0.2500,0.3636)	(0.4250,0.5250,0.6500)
A_3	(0.7727,0.8864,1.0000)	(0.7222,0.8611,1.0000)	(0.4000,0.5000,0.6667)	(0.4318,0.5227,0.6364)	(0.4000,0.4750,0.6000)
A_4	(0.2045,0.2500,0.3636)	(0.5833,0.6944,0.8333)	(0.6333,0.7667,0.9333)	(0.5455,0.6591,0.7727)	(0.5750,0.6750,0.8000)
A_5	(0.4545,0.5682,0.6818)	(0.4167,0.5278,0.6667)	(0.4333,0.5667,0.7333)	(0.7727,0.8864,1.0000)	(0.5750,0.6750,0.8000)

Table 16. The normalized aggregate fuzzy decision matrix Y_{WAM} obtained from the matrix \tilde{X}_{WAM}

Criteria	C_1	C_2	C_3	C_4	C_5
A_1	(0.6452,0.7604,0.8756)	(0.3519,0.4501,0.5926)	(0.4486,0.5426,0.7199)	(0.3230,0.4336,0.5442)	(0.7449,0.8724,1.0000)
A_2	(0.4263,0.5415,0.6567)	(0.6781,0.7892,0.9316)	(0.6454,0.8227,1.0000)	(0.1748,0.2124,0.3230)	(0.4184,0.5077,0.6352)
A_3	(0.7696,0.8848,1.0000)	(0.7151,0.8575,1.0000)	(0.3883,0.4787,0.6560)	(0.4115,0.4978,0.6084)	(0.4082,0.4898,0.6173)
A_4	(0.2108,0.2558,0.3710)	(0.5513,0.6695,0.8120)	(0.6773,0.8156,0.9929)	(0.5575,0.6681,0.7788)	(0.5816,0.6811,0.8087)
A_5	(0.4954,0.6106,0.7258)	(0.3932,0.4929,0.6353)	(0.4805,0.6277,0.8050)	(0.7788,0.8894,1.0000)	(0.5293,0.6327,0.7602)

Table 17. The normalized aggregate fuzzy decision matrix Y_{PK} obtained from the matrix \tilde{X}_{PK}

Criteria	C_1	C_2	C_3	C_4	C_5
A_1	(0.4000,0.6600,1.0000)	(0.1000,0.3000,0.6000)	(0.1250,0.4250,1.0000)	(0.2000,0.4200,0.8000)	(0.4000,0.7000,1.0000)
A_2	(0.2000,0.5000,0.8000)	(0.1000,0.5400,1.0000)	(0.2500,0.6250,1.0000)	(0.1000,0.2200,0.6000)	(0.1000,0.4200,1.0000)
A_3	(0.2000,0.7800,1.0000)	(0.4000,0.6200,0.8000)	(0.1250,0.3750,0.7500)	(0.1000,0.4600,0.8000)	(0.1000,0.3800,1.0000)
A_4	(0.1000,0.2200,0.6000)	(0.1000,0.5000,1.0000)	(0.1250,0.5750,1.0000)	(0.4000,0.5800,0.8000)	(0.1000,0.5400,0.8000)
A_5	(0.2000,0.5000,0.8000)	(0.1000,0.3800,0.4800)	(0.1250,0.4250,1.0000)	(0.6000,0.7800,1.0000)	(0.1000,0.5400,1.0000)

Table 18. The normalized aggregate fuzzy decision matrix Y_{WPK} obtained from the matrix \tilde{X}_{WPK}

Criteria	C_1	C_2	C_3	C_4	C_5
A_1	(0.1600,2.2000,0.7333)	(0.0500,1.3167,0.5500)	(0.0944,1.7000,0.8444)	(0.1133,1.3067,0.4400)	(0.3000,2.8500,0.9167)
A_2	(0.1267,1.5667,0.5867)	(0.0917,2.3083,1.0000)	(0.3333,2.5778,0.7333)	(0.0567,0.6400,0.2933)	(0.1000,1.6583,0.7917)
A_3	(0.1467,2.5600,1.0000)	(0.3000,2.5083,0.7500)	(0.1056,1.5000,0.7333)	(0.0733,1.5000,0.6000)	(0.0708,1.6000,0.9167)
A_4	(0.0400,0.7400,0.4400)	(0.0708,1.9583,0.9167)	(0.1222,2.5556,1.0000)	(0.1600,2.0133,0.8000)	(0.0917,2.2250,1.0000)
A_5	(0.0800,1.7667,0.8000)	(0.1250,1.4417,0.5500)	(0.0944,1.9667,0.9778)	(0.2400,2.6800,1.0000)	(0.0792,2.0667,0.7500)



For each matrix (Table 15-18) we have the following positive-ideal and negative-ideal solutions, respectively:

$$\begin{aligned}
 A_{AM}^+ &= (1.0000, 1.0000, 1.0000, 1.0000, 1.0000), \\
 A_{AM}^- &= (0.2045, 0.3333, 0.4000, 0.2045, 0.4000), \\
 A_{WAM}^+ &= (1.0000, 1.0000, 1.0000, 1.0000, 1.0000), \\
 A_{WAM}^- &= (0.2108, 0.3519, 0.3883, 0.1748, 0.4082) \\
 A_{PK}^+ &= (1.0000, 1.0000, 1.0000, 1.0000, 1.0000), \\
 A_{PK}^- &= (0.1000, 0.1000, 0.1250, 0.1000, 0.1000) \\
 A_{WPK}^+ &= (1.0000, 1.0000, 1.0000, 1.0000, 1.0000), \\
 A_{WPK}^- &= (0.0400, 0.0500, 0.0944, 0.0567, 0.0708)
 \end{aligned}$$

Finally, using the Tables 12 and 14, and Eqs.(19), (22)-(25), the closeness indexes which are calculated to determine the ranking order of all alternatives are given in Table 19. The notations used in Table 19 can be interpreted as follows:

- “Weight: Entropy+AM” shows that for each decision maker the weight of criteria is calculated by applying the entropy method and the average weight is calculated using the aggregation operator AM (Eq.(34)). And “Matrix: AM” shows that the group fuzzy decision matrix is obtained by the AM operator (Eq.(6));
- “Weight: Entropy+AM” – for each decision maker the weight of criteria is calculated by the entropy method and the average weight is calculated using the aggregation operator AM; “Matrix: WAM” – the group fuzzy decision matrix is obtained by WAM operator (Eq.(7));
- “Weight: Entropy+WAM” – for each decision maker the weight of criteria is calculated by applying the entropy method and the average weight is calculated using the aggregation operator WAM (Eq.(35)); “Matrix: AM” – the matrices are aggregated by the AM operator;
- “Weight: Entropy+WAM” – the criteria weights are calculated by the Entropy method and aggregated by the WAM operator; “Matrix: WAM” – the matrices are aggregated by the WAM operator;
- “Weight: WC” – the criteria weights are calculated by the Worst-Case method (Eqs.(31) and (33)); “Matrix: AM” – the matrices are aggregated by the AM operator;
- “Weight: WC” – the criteria weights are calculated by the Worst-Case method; “Matrix: WAM” – the matrices are aggregated by the WAM operator;
- “Weight: Entropy+AM” – the criteria weights are calculated by the Entropy method and aggregated by AM operator; “Matrix: PK” – the matrices are aggregated by the PK operator (Eq.(4));
- “Weight: Entropy+AM” – the criteria weights are calculated by the Entropy method and aggregated by AM operator; “Matrix: WPK” – the matrices are aggregated by the WPK operator (Eq.(5));
- “Weight: Entropy+WAM” – the criteria weights are calculated by the Entropy method and aggregated by WAM operator; “Matrix: PK” – the matrices are aggregated by the PK operator;
- “Weight: Entropy+WAM” – the criteria weights are calculated by the Entropy method and aggregated by WAM operator; “Matrix: WPK” – the matrices are aggregated by the WPK operator;
- “Weight: WC” – the criteria weights are calculated by the Worst-Case method; “Matrix: PK” – the matrices are aggregated by the PK operator;
- “Weight: WC” – the criteria weights are calculated by the Worst-Case method; “Matrix: WPK” – the matrices are aggregated by the WPK operator.



Table 19. The ranks of the alternatives obtained by different methods using aggregation of matrices

Alternatives	1		2		3		4		5		6	
	Weight: Entropy+AM Matrix: AM		Weight: Entropy+AM Matrix: WAM		Weight: Entropy+WAM Matrix: AM		Weight: Entropy+WAM Matrix: WAM		Weight: WC Matrix: AM		Weight: WC Matrix: WAM	
	CI	Rank	CI	Rank	CI	Rank	CI	Rank	CI	Rank	CI	Rank
A_1	0.4743	5	0.4531	4	0.4747	5	0.4533	4	0.4946	4	0.4714	4
A_2	0.4780	4	0.4218	5	0.4789	4	0.4223	5	0.4800	5	0.4173	5
A_3	0.5174	1	0.5015	2	0.5164	2	0.5004	2	0.4966	3	0.4805	2
A_4	0.4974	3	0.4760	3	0.4981	3	0.4765	3	0.5002	2	0.4779	3
A_5	0.5173	2	0.5184	1	0.5170	1	0.5180	1	0.5152	1	0.5148	1
Alternatives	7		8		9		10		11		12	
	Weight: Entropy+AM Matrix: PK		Weight: Entropy+AM Matrix: WPK		Weight: Entropy+WAM Matrix: PK		Weight: Entropy+WAM Matrix: WPK		Weight: WC Matrix: PK		Weight: WC Matrix: WPK	
	CI	Rank	CI	Rank	CI	Rank	CI	Rank	CI	Rank	CI	Rank
A_1	0.5312	1	0.5894	4	0.5318	1	0.5895	4	0.5458	1	0.5914	4
A_2	0.5083	4	0.5837	5	0.5091	4	0.5838	5	0.5144	3	0.5851	5
A_3	0.5013	5	0.6003	2	0.5012	5	0.6003	2	0.4984	5	0.6005	2
A_4	0.5091	3	0.5932	3	0.5097	3	0.5932	3	0.5113	4	0.5933	3
A_5	0.5284	2	0.6008	1	0.5289	2	0.6009	1	0.5346	2	0.6012	1

As it is seen from the Table 19, the method of calculating the criteria weights, the averaging strategies and importance value of decision makers influence on evaluation of alternatives (their ranks). For example, by comparing the 1st and 2nd columns, we can see that, in spite of the calculation and aggregation of criteria weights in the same way in both cases, the ranks of alternatives differ from each others because of the matrices are aggregated by different methods (in the 1st case the group decision matrix is calculated by the AM operator and in the 2nd case by the WAM operator). If A_3 is selected the best alternative in the 1st case, A_5 is the best alternative in the 2nd case. In the same way, the differences can be seen by comparing the others. Kendall's rank correlation (Kendall & Gibbons, 1990) has been used to show these differences below.

Let's calculate Kendall's rank correlation coefficient among the ranks given in 1st and 5th, 2nd and 6th, 7th and 11th, 8th and 12th columns in the Table 19 in order to show the influence of the method of calculating the criteria weights on evaluation of the alternatives (for the sake of simplicity, we have numbered the columns of this Table from 1 to 12):

$$\tau(1,5) = 0.4; \tau(2,6) = 1.0; \tau(7,11) = 0.8; \tau(8,12) = 1.0.$$

It is seen that, if matrices are aggregated by taking into account of the importance values of decision makers, then the method of calculation of criteria weights does not have any influence on the evaluation. Because of this, we need to pay attention to the values $\tau(1, 5)$ vs $\tau(2, 6)$, $\tau(7, 11)$ vs $\tau(8, 12)$. Indeed, if we compare 1st and 2nd; 5th and 6th columns, we can see that they differ only by the matrix aggregation method. This also confirms the $\tau(2, 6) = \tau(8, 12) = 1$ values.

Let's calculate Kendall's rank correlation coefficient between the corresponding columns in Table 19 in order to see the influence of matrix aggregation method on the ranking of alternatives:

$$\begin{aligned} \tau(1, 2) &= 0.6; \tau(1, 7) = -0.4; \tau(1, 8) = 0.6; \tau(2, 7) = 0.0; \tau(2, 8) = 1.0; \\ \tau(7, 8) &= 0.0; \tau(3, 4) = 0.8; \tau(3, 9) = -0.2; \tau(3, 10) = 0.8; \tau(4, 9) = 0.0 \\ \tau(4, 10) &= 1.0; \tau(9, 10) = 0.0; \tau(5, 6) = 0.8; \tau(5, 11) = 0.0; \tau(5, 12) = 0.8; \\ \tau(6, 11) &= -0.2; \tau(6, 12) = 1.0; \tau(11, 12) = -0.2. \end{aligned}$$

As it is seen above, PK method poorly correlated with other methods:

$$\tau(1, 7) = -0.4; \tau(3, 9) = -0.2; \tau(6, 11) = -0.2; \tau(2, 7) = 0.0; \tau(4, 9) = 0.0; \tau(11, 12) = -0.2$$

The similar pattern that we observed above is demonstrated here too. Thus, if the importance values of decision makers are taken into account in aggregation of matrices then influence of



method calculation of the criteria weights on evaluation of alternatives is minimized again. For example, $\tau(1, 7) = -0.4$; $\tau(3, 9) = -0.2$, $\tau(6,11) = -0.2$, where $\tau(1,8) = 0.6$; $\tau(3,10) = 0.8$; $\tau(6,12) = 1.0$. Unlike PK method, WPK method does not have any influence on evaluation:

$$\tau(2, 8) = 1.0; \tau(4, 10) = 1.0; \tau(6, 12) = 1.0.$$

As it is seen, the matrices are aggregated by any operator in the case of above evaluation process, then the positive-ideal and negative-ideal solutions are found for aggregate matrix. In the result, the closeness index of alternatives to positive-ideal solution is calculated. It is noted above, evaluation of alternatives can be carried out by other method, that is, without aggregation of the matrices. In this case, positive-ideal and negative-ideal solutions are found for each decision matrix, then the weighted Euclidean distances between the alternatives and these solutions are calculated (Eqs.(22), (23)). Then these distances are aggregated (Eqs.(27), (28)) and finally, the closeness index for each alternative is calculated (Eq.(29)). The evaluation result of alternatives by the aggregation of distances is given in Table 20.

Table 20. The ranks of the alternatives obtained by aggregation of distances

Alternatives	13		14		15		16		17		18	
	Weight: Entropy+AM Distance: AM		Weight: Entropy+AM Distance: WAM		Weight: Entropy+WAM Distance: AM		Weight: Entropy+WAM Distance: WAM		Weight: WC Distance: AM		Weight: WC Distance: WAM	
	CI	Rank	CI	Rank	CI	Rank	CI	Rank	CI	Rank	CI	Rank
A ₁	0.4447	4	0.4422	4	0.4448	4	0.4422	4	0.4601	3	0.4568	4
A ₂	0.4330	5	0.4236	5	0.4332	5	0.4238	5	0.4328	5	0.4226	5
A ₃	0.4741	2	0.4705	3	0.4732	2	0.4696	3	0.4600	4	0.4578	3
A ₄	0.4763	1	0.4830	1	0.4766	1	0.4834	1	0.4773	1	0.4854	1
A ₅	0.4685	3	0.4814	2	0.4681	3	0.4810	2	0.4667	2	0.4789	2

We have noted above that, one of the aims of this study is to show the difference between the results obtained by the aggregation of matrices and the aggregation of distances. Let's calculate Kendall's rank correlation coefficient between the corresponding columns in Table 19 and 20 to show this difference: $\tau(1,13)=0.4$; $\tau(2,14)=0.6$; $\tau(3,15)=0.2$; $\tau(4,16)=0.6$; $\tau(5,17)=0.6$; $\tau(6,18)=0.6$.

The question may arise that what is the corresponding columns? As it is seen, the criteria weights were calculated by the same method in the relative columns (1st and 13rd; 2nd and 14th; 3rd and 15th; and etc.). The matrices and distances were aggregated by the same operator. It is seen from correlation coefficient that, such approaches (the aggregation of matrices and the aggregation of distances) do not demonstrate the same results. As in the first approach (the aggregation of matrices), the calculation of the criteria weights and the aggregation methods of distances directly affects to the evaluation process (ranking of alternatives).

All these demonstrate that making a decision based only on single method in evaluation of alternatives is not so effective approach. This may create doubt for faithfulness of the result. That is why, the evaluation should be conducted using several methods and the results should be aggregated. The aggregation of results can be conducted by various methods. In this study, we use the following formula (Aliguliyev, 2009) to aggregate the results obtained by different methods:

$$RR(A_i) = \sum_{s=1}^p \frac{(p-s+1)}{p} r_s. \tag{40}$$

Where r_s denotes the number of times the alternative appears in the s th rank; p is the number of evaluation methods; $RR(A_i)$ is the resultant rank of the alternative A_i . From Tables 19 and 20, by applying this formula, we obtain Tables 21 and 22.

Table 21. The aggregate ranks of the alternatives obtained from Table 19 using Eq.(40)

Alternatives	The number of times the alternative is in the s th rank, $s=$					Resultant Rank, RR	Aggregate rank, AR_M
	1	2	3	4	5		
A ₁	0	0	0	4	2	2.0	4
A ₂	0	0	0	2	4	1.6	5
A ₃	1	4	1	0	0	4.8	2
A ₄	0	1	5	0	0	3.8	3
A ₅	5	1	0	0	0	5.8	1



Table 22. The aggregate ranks of the alternatives obtained from Table 20 using Eq.(40)

Alternatives	The number of times the alternative is in the <i>s</i> th rank, <i>s</i> =					Resultant Rank, <i>RR</i>	Aggregate rank, <i>AR_D</i>
	1	2	3	4	5		
<i>A₁</i>	0	0	1	5	0	2.6	4
<i>A₂</i>	0	0	0	0	6	1.2	5
<i>A₃</i>	0	2	3	1	0	3.8	3
<i>A₄</i>	6	0	0	0	0	6.0	1
<i>A₅</i>	0	4	2	0	0	4.4	2

Analysis shows that aggregation of the ranks by this method may yield the expected result. Indeed, the Aggregate Rank (*AR_M*) shown in the Table 21 (last column) good correlated with the ranks obtained by the different methods reported in Table 19. Here the only exceptions are 7th, 9th and 11th columns. There also has a regularity. Thus, as we noted above, the ranks that are obtained by the aggregation of matrices using the PK method poorly correlated with each of other methods. In spite of this, *AR_M* minimizes the influence of PK method. Indeed, despite the correlation coefficient between the ranks specified in 1st and 7th columns equals $\tau(1,7) = -0.4$, however, *AR* well correlated with each ranks specified in the 1st and 7th columns:

$$\tau(AR_M, 1) = 0.6; \quad \tau(AR_M, 7) = 0.0.$$

It is possible to see the same result by calculating correlation coefficient between Aggregate Ranks, *AR_M* and *AR_D*, given in Table 21 and 22:

$$\tau(AR_M, AR_D) = 0.6$$

At the end, merging of the Table 21 and 22 is proposed to minimize the all possible effects. According to the aggregate rank (*AR*), ranking the preference of these alternatives is as Table 23.

Table 23. The aggregate rank of the alternatives obtained by aggregation of the ranks *AR_M* and *AR_D*

Alternatives	The number of times the alternative is in the <i>s</i> th rank,					Resultant Rank	Aggregate Rank, <i>AR</i>
	1	2	3	4	5		
<i>A₁</i>	0	0	1	9	2	4.6	4
<i>A₂</i>	0	0	0	2	10	2.8	5
<i>A₃</i>	1	6	4	1	6	8.6	3
<i>A₄</i>	6	1	5	0	0	9.8	2
<i>A₅</i>	5	5	2	0	0	10.2	1

The alternatives, hence, were ranked according to the aggregate rank (*AR*) (Table 23). According to the multi-criteria evaluation, the fifth alternative (*A₅*) should be recruited, whereas the fourth alternative (*A₄*) is the second-best option. At the other ends of spectrum, alternatives *A₃*, *A₁* and *A₂* are the last three. The relative relational degree of alternatives is determined, and then five alternatives are ranked as $A_5 > A_4 > A_3 > A_1 > A_2$.

It can be easily see that the rank *AR* obtained by this method good correlated with the ranks *AR_M* and *AR_D* obtained by other two methods (Tables 22 and 23):

$$\tau(AR, AR_M) = 0.8; \quad \tau(AR, AR_D) = 0.8.$$

In our opinion, the evaluation of alternatives by this method is the best approach. When we compare *AR* that given in Table 23 with the ranks obtained by different methods demonstrated in Tables 19 and 20, it can be seen that, $\tau(5, AR) = 1.0$. That is, the resultant rank gives the same result with the result obtained by the method that shown in the 5th column of Table 19, where the criteria weights are calculated by WC method, while the matrices are aggregated by AM operator. It is a bit difficult to put forward an idea about why it is so. It demands a new research.

5. CONCLUSION

MCDM has been widely used in the solution of real word decision making problems. By considering the fact that, in some cases, determining precisely the exact values of alternatives with respect to the criteria or/and the exact values for the weights of criteria, is difficult or impossible. Then, the values of alternatives with respect to the criteria or/and the values of criteria weights are considered as fuzzy values. So the conventional approaches for solving these MCDM problems tend to be less effective in dealing with the imprecise or vagueness nature of the linguistic



assessment. In such conditions, the fuzzy MCDM methods are applied for solving fuzzy MCDM problems. To address the disadvantages of traditional personnel evaluation methods, this paper proposed the use of a hybrid fuzzy group decision making method. In particular, this paper proposed hybrid fuzzy TOPSIS+WC and TOPSIS+ENTROPY methods. It is known that in group decision settings, different fuzzy group MCDM methods often produce inconsistent ranking outcomes for the same problem. Despite the importance of validating the ranking outcomes produced by different methods, very few studies have been conducted to help a group of decision makers' deal with the ranking inconsistency problem produced by different methods. To address the ranking inconsistency problem in fuzzy group MCDM, this paper developed a new approach for fusion of the ranks obtained by different MCDM methods. This approach produced the most preferred group ranking outcome for a given problem. Based on three averaging operators and two methods for definition of the criteria weights 12 fuzzy group MCDM evaluation strategies are developed as an illustration to solve the general fuzzy MCDM problem that requires cardinal ranking of the alternatives. An empirical study on the information personnel evaluation problem is used to illustrate how the approach works. With its simplicity in both concept and computation, the approach can be applied in general fuzzy group decision problems solvable by many fuzzy group MCDM methods. It is particularly suited to large-scale fuzzy group MCDM problems where the ranking outcomes produced by different methods differ significantly. Further studies should focus on development of the weighted hybrid MCDM method to solve the ranking inconsistency problem in fuzzy group MCDM.

REFERENCES

- Alguliyev, R., & Mahmudova, R. (2011). Structural approach to the formation of information culture of individuals. In: *Proceedings of the International Conference on Informatics Engineering and Information Science*, Kuala Lumpur, Malaysia, part IV, vol.254, (pp.29-40).
- Aliguliyev, R. (2009). Performance evaluation of density-based clustering methods. *Information Sciences*, 179(20), 3583-3602.
- Balezentis, A., Balezentis, T., & Brauers, W.K.M. (2012). Personnel selection based on computing with words and fuzzy MULTIMOORA. *Expert Systems with Applications*, 39(9), 7961-7967.
- Balezentis, T., & Zeng, S. (2013). Group multi-criteria decision making based upon interval-valued fuzzy numbers: an extension of the MULTIMOORA method. *Expert Systems with Applications*, 40(2), 543-550.
- Bergeron, P., Heaton, L., Choo, C.W., Detlor, B., Bouchard, D., & Paquette, S. (2007). Knowledge and information management practices in knowledge-intensive organizations: a case study of a Quebec public health management organization. In: *Proceedings of the 35th Annual Conference Canadian Association for Information Science*, Montreal, Quebec. Retrieved 31 March, 2014 from http://www.cais-acsi.ca/proceedings/2007/bergeron_2007.pdf.
- Buyukozkan, G., & Cifci, G. (2012). A combined fuzzy AHP and fuzzy TOPSIS based strategic analysis of electronic service quality in healthcare industry. *Expert Systems with Applications*, 39(3), 2341-2354.
- Cameron, K. S., & Ettington, D. R. (1988). The conceptual foundations of organizational culture. *Higher Education: Handbook of Theory and Research*, 4, 429-447.
- Cameron, K.S., & Freeman, S.J. (1991). Cultural congruence, strength, and type: Relationships to effectiveness. *Research in Organizational Change and Development*, 5, 23-58.
- Cameron, K.S., & Quinn, R.E. (2011). *Diagnosing and changing organizational culture: based on the competing values framework*. Reading, MA: Jossey Bass.
- Chang, Y.-H., Yeh, C.-H., & Chang, Y.-W. (2013). A new method selection approach for fuzzy group multicriteria decision making. *Applied Soft Computing*, 13(4), 2179-2187.
- Chien, C.-F., & Chen, L.-F. (2008). Data mining to improve personnel selection and enhance human capital: a case study in high-technology industry. *Expert Systems with Applications*, 34(1), 280-290.



- Choo, C.W. (2013). Information culture and organizational effectiveness. *International Journal of Information Management*, 33(5), 775–779.
- Choo, C.W., Bergeron, P., Detlor, B., & Heaton, L. (2008). Information culture and information use: an exploratory study of three organizations. *Journal of the American Society for Information Science and Technology*, 59(5), 792-804.
- Choo, C.W., Furness, C., Paquette, S., van den Berg, H., Detlor, B., Bergeron, P., & Heaton, L. (2006). Working with information: information management and culture in a professional services organization. *Journal of Information Science*, 32(6), 491-510.
- Curry, A., & Moore, C. (2003). Assessing information culture – an exploratory model. *International Journal of Information Management*, 23, 91-110.
- Dagdeviren, M. (2010). A hybrid multi-criteria decision-making model for personnel selection in manufacturing systems. *Journal of Intelligent Manufacturing*, 21(4), 451-460.
- Deng, H., & Yeh, C.-H. (2006). Simulation-based evaluation of defuzzification-based approaches to fuzzy multiattribute decision making. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 36(5), 968-977.
- Deng, Y., Chan, F.T.S., Wu, Y., & Wang, D. (2011). A new linguistic MCDM method based on multiple-criterion data fusion. *Expert Systems with Applications*, 38(6), 6985-6993.
- Dursun, M., & Karsak, E.E. (2010). A fuzzy MCDM approach for personnel selection. *Expert Systems with Applications*, 37(6), 4324-4330.
- Ginevicius, R. (2011). A new determining method for the criteria weights in multicriteria evaluation. *International Journal of Information Technology and Decision Making*, 10(6), 1067-1095.
- Guo, J. (2013). Hybrid multiattribute group decision making based on intuitionistic fuzzy information and GRA method. *ISRN Applied Mathematics*, 2013, Article ID 146026, 10 pp.
- Güngör, Z., Serhadlıoğlu, G., & Kesen, S.E. (2009). A fuzzy AHP approach to personnel selection problem. *Applied Soft Computing*, 9(2), 641–646.
- Hwang, C.L., & Yoon, K. (1981). Multiple attribute decision making: methods and applications. *Lecture Notes in Economics and Mathematical Systems*, 186.
- Jahanshahloo, G.R., Lotfi, F.H., & Izadikhah, M. (2006). Extension of the TOPSIS method for decision-making problems with fuzzy data. *Applied Mathematics and Computation*, 181(2), 1544-1551.
- Karsak, E.E. (2001). Personnel selection using a fuzzy MCDM approach based on ideal and anti-ideal solutions. *Lecture Notes in Economics and Mathematical Systems*, 507, 393-402.
- Kelemenis, A., & Askounis, D. (2010). A new TOPSIS-based multi-criteria approach to personnel selection. *Expert Systems with Applications*, 37(7), 4999-5008.
- Kendall, M., & Gibbons, J.D. (1990). Rank correlation methods (5th ed.). Edward Arnold, London.
- Khan, M.N., & Azmi, F.T. (2005). Reinventing business organizations: the information culture framework. *Singapore Management Review*, 27(2), 37-63.
- Liang, G.S., & Wang, M.J.J. (1994). Personnel selection using fuzzy MCDM algorithm. *European Journal of Operational Research*, 78(1), 22-33.
- Lin, H.T. (2010). Personnel selection using analytic network process and fuzzy data envelopment analysis approaches. *Computers & Industrial Engineering*, 59(4), 937-944.
- Noor-E-Alama, Md., Lipi, T.F., Hasina, M.A.A., & Ullah, A.M.M.S. (2011). Algorithms for fuzzy multi expert multi criteria decision making (ME-MCDM). *Knowledge-Based Systems*, 24(3), 367–377.
- Oostrom, J.K., van der Linden, D., Born, M.Ph., & van der Molen, H.T. (2013). New technology in personnel selection: how recruiter characteristics affect the adoption of new selection technology. *Computers in Human Behavior*, 29(6), 2404-2415.



- Paksoy, T., Pehlivan, N.Y., & Kahraman, C. (2012). Organizational strategy development in distribution channel management using fuzzy AHP and hierarchical fuzzy TOPSIS. *Expert Systems with Applications*, 39(3), 2822–2841.
- Patil, S.K., & Kant, R. (2014). A fuzzy AHP-TOPSIS framework for ranking the solutions of knowledge management adoption in supply chain to overcome its barriers. *Expert Systems with Applications*, 41(1), 679-693.
- Rao, R.V. (2013). Decision making in the manufacturing environment using graph theory and fuzzy multiple attribute decision making methods. *Springer Series in Advanced Manufacturing*, Springer-Verlag, London, 2013, 292pp.
- Rotshtein, A.P. (2009). Fuzzy multicriteria choice among alternatives: worst-case approach. *Journal of Computer and Systems Sciences International*, 48(3), 379-383.
- Saaty, T.L. (2006). *Fundamentals of decision making and priority theory with the Analytic Hierarchy Process*. RWS Publications, Pittsburgh, 2006, 477pp.
- Saaty, T.L. (2008). Decision making with the analytic hierarchy process. *International Journal of Services Sciences*, 1(1), 83-98.
- Sangdee, P., Apichatvullop, Y., & Manmart, L. (2009). Investigating information culture in Thai public universities. In: *Proceedings of the 14th Congress of Southeast Asian Librarians*, Hanoi, Vietnam, (pp.203-216).
- Shyur, H.-J., & Shih, H.-S. (2006). A hybrid MCDM model for strategic vendor selection. *Mathematical and Computer Modelling*, 44(7-8), 749-761.
- Steinwachs, K. (1999). Information and culture: the impact of national culture on information processes. *Journal of Information Science*, 25 (3), 193-204.
- Su, Z.-X. (2011). A hybrid fuzzy approach to fuzzy multi-attribute group decision-making. *International Journal of Information Technology & Decision Making*, 10(4), 695-711.
- Wan, S.-P., Wang, Q.-Y., & Dong, J.-Y. (2013). The extended VIKOR method for multi-attribute group decision making with triangular intuitionistic fuzzy numbers. *Knowledge-Based Systems*, 52, 65-77.
- Wu, Y. (2010). Using ANP to do the information personnel evaluation. *Key Engineering Materials*, 439-440, 749-753.
- Yu, D., Zhang, W., & Xu, Y. (2013). Group decision making under hesitant fuzzy environment with application to personnel evaluation. *Knowledge-Based Systems*, 52, 1-10.
- Zhang, S.-F., & Liu, S.-Y. (2011). A GRA-based intuitionistic fuzzy multi-criteria group decision making method for personnel selection. *Expert Systems with Applications*, 38(9), 11401-11415.
- Zhang, W., & Zhang, Q. (2014). Multi-stage evaluation and selection in the formation process of complex creative solution. *Quality & Quantity: International Journal of Methodology*, 48(5), 2375-2404.

